ABOUT PLANAR \((\alpha, \beta)\)-ACCESSIBLE DOMAINS

Abstract. The article is devoted to the class \(A^{\alpha,\beta}_\rho\) of all \((\alpha, \beta)\)-accessible with respect to the origin domains \(D\), \(\alpha, \beta \in [0, 1]\), possessing the property \(\rho = \min_{p \in \partial D} |p|\), where \(\rho \in (0, +\infty)\) is a fixed number. We find the maximal set of points \(a\) such that all domains \(D \in A^{\alpha,\beta}_\rho\) are \((\gamma, \delta)\)-accessible with respect to \(a\), \(\gamma \in [0; \alpha]\), \(\delta \in [0; \beta]\). This set is proved to be the closed disc of center 0 and radius \(\rho \sin \frac{\varphi \pi}{2}\), where \(\varphi = \min \{\alpha - \gamma, \beta - \delta\}\).

Key words: \(\alpha\)-accessible domain, \((\alpha, \beta)\)-accessible domain, cone condition

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In [1] the notion of \(\alpha\)-accessible domain was introduced. Let \(\alpha \in [0, 1)\) be a fixed number. A domain \(D \subset \mathbb{R}^n\), \(0 \in D\), is called \(\alpha\)-accessible if for every point \(p \in \partial D\) there exists a number \(r = r(p) > 0\) such that the cone

\[
K_+(p, \alpha, r) = \left\{ x \in \mathbb{R}^n : \|x\| \leq r, \left( x - p, \frac{p}{\|p\|} \right) \geq \|x - p\| \cos \frac{\alpha \pi}{2} \right\}
\]

is included in \(\mathbb{R}^n \setminus D\).

In the case \(n = 2\) these domains have been studied earlier by J. Stankiewicz [2—4], D. A. Brannan and W. E. Kirwan [5], W. Ma and D. Minda [6], T. Sugawa [7] and others as a generalization of starlike domains. It was noted (see, for example, [7—10]) that in the planar case it is possible to consider domains that possess a more general property. In [9, 10] such domains were called \((\alpha, \beta)\)-accessible.

Definition 1. [9], [10] Let \(\alpha, \beta \in [0, 1]\), \(D \subset \mathbb{C}\), \(a \in D\). A domain \(D\) is called \((\alpha, \beta)\)-accessible with respect to \(a\) if for every point \(p \in \partial D\) there
exists a number \( r = r(p) > 0 \) such that the cone
\[
K_+(p, a, \alpha, \beta, r) = \left\{ z \in \mathbb{C} : -\frac{\beta \pi}{2} \leq \arg(z - p) - \arg(p - a) \leq \frac{\alpha \pi}{2}, |z - p| \leq r \right\}
\]
is contained in \( \mathbb{C} \setminus D \).

The notion of \((\alpha, \beta)\)-accessible domain is a generalization of the notion of \(\alpha\)-accessible domain. They are equal if \( \alpha = \beta \).

In the article we choose values of arguments so that their difference belongs to \((-\pi; \pi]\).

It was shown in \([1]\) and \([9, 10]\) that \(\alpha\)- and \((\alpha, \beta)\)-accessible domains satisfy the so-called “cone condition”, i.e. such domains are also conically accessible from the interior.

The problem of characterization all domains with the “cone condition” is very hard. \(\alpha\)- and \((\alpha, \beta)\)-accessible domains are only special but important cases of such domains. For \(\alpha\)-accessible domains the axis of symmetry of the cone is radial, for \((\alpha, \beta)\)-accessible domains \(D\) the angle between this axis and the vector \( p - a, p \in \partial D\), is fixed.

The following criterion of \((\alpha, \beta)\)-accessibility is needed for the sequel.

**Theorem A.** \([9, 10]\) Let \( D \subset \mathbb{C}, \partial D \) be smooth, \( n(p) \) be an outward normal to the domain \( D \) at a point \( p \in \partial D \), \( \alpha, \beta \in (0; 1) \). Then the domain \( D \) is \((\alpha, \beta)\)-accessible with respect to the origin if and only if
\[
-\frac{(1 - \beta) \pi}{2} \leq \arg(p) - \arg(n(p)) \leq \frac{(1 - \alpha) \pi}{2}
\]
for every \( p \in \partial D \).

Let us note that Theorem A can be applied locally. In particular, if \( \Gamma \) is an open smooth curve, \( \Gamma \subset \partial D \), a point \( p \in \Gamma \), then from the proof of Theorem A it follows that the unbounded cone
\[
K_+(p, 0, \alpha, \beta) = \left\{ z \in \mathbb{C} : -\frac{\beta \pi}{2} \leq \arg(z - p) - \arg p \leq \frac{\alpha \pi}{2} \right\}
\]
is included in \( \mathbb{C} \setminus D \).

Consider the class \( A_{\rho}^{\alpha, \beta} \), containing all \((\alpha, \beta)\)-accessible with respect to the origin domains \( D \), possessing the property \( \min_{p \in \partial D} |p| = \rho \), where
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\(\rho \in (0, \infty)\) is a fixed number. Let \(\gamma \in [0; \alpha]\), \(\delta \in [0; \beta]\), \(D \in A^\alpha_{\rho, \beta}\). By \(\Omega_D^{\gamma, \delta}\) denote the set of all points \(a \in D\) such that the domain \(D\) is \((\gamma, \delta)\)-accessible with respect to \(a\).

In the present paper we find the maximal set that is contained in \(\Omega_D^{\gamma, \delta}\) for all domains \(D\) from \(A^\alpha_{\rho, \beta}\).

By \(\mathbb{B}[0, R]\) denote the disc \(\{z \in \mathbb{C} : |z| \leq R\}\).

**Theorem 1.** If \(\alpha, \beta \in [0; 1)\), \(\gamma \in [0; \alpha]\), \(\delta \in [0; \beta]\) then

\[
\bigcap_{D \in A^\alpha_{\rho, \beta}} \Omega_D^{\gamma, \delta} = \mathbb{B} \left[ 0, \rho \sin \frac{\varphi \pi}{2} \right],
\]

where \(\varphi = \min \{\alpha - \gamma, \beta - \delta\}\).

**Proof.** Let us show that the set \(\bigcap_{D \in A^\alpha_{\rho, \beta}} \Omega_D^{\gamma, \delta}\) is a disc. If \(D \in A^\alpha_{\rho, \beta}\), then a domain \(U(D)\), obtained by rotation of \(D\) with respect to the origin, also belongs to \(A^\alpha_{\rho, \beta}\). Therefore the set \(\bigcap_{U} \Omega_{U(D)}^{\gamma, \delta}\), where the intersection extends over all rotation transformations of \(D\), is a disc with center at the origin. Hence,

\[
\bigcap_{D \in A^\alpha_{\rho, \beta}} \Omega_D^{\gamma, \delta} = \bigcap_{D \in A^\alpha_{\rho, \beta}} \bigcap_{U} \Omega_{U(D)}^{\gamma, \delta}
\]

is a disc too.

Let \(\alpha, \beta \in [0; 1)\), \(D \in A^\alpha_{\rho, \beta}\). Fix \(p \in \partial D\). Since the domain \(D\) is \((\alpha, \beta)\)-accessible with respect to the origin, then \(K_+(p, 0, \alpha, \beta, r) \subset \mathbb{C} \setminus D\) for some \(r > 0\). Let us show that for all points \(a\) from the intersection of the domain \(D\) and the cone

\[
K_-(p, 0, \alpha - \gamma, \beta - \delta) = \left\{ z \in \mathbb{C} : -\frac{(\beta - \delta)\pi}{2} \leq \arg(z - p) - \arg(-p) \leq \frac{(\alpha - \gamma)\pi}{2} \right\}
\]

the following inclusion holds

\[
K_+(p, a, \gamma, \delta, r) \subset K_+(p, 0, \alpha, \beta, r).
\]

Take \(a \in K_-(p, 0, \alpha - \gamma, \beta - \delta) \cap D\), then

\[
-\frac{(\beta - \delta)\pi}{2} \leq \arg(a - p) - \arg(-p) \leq \frac{(\alpha - \gamma)\pi}{2}.
\] (2)
Let \( z \in K_+(p, a, \gamma, \delta, r), z \neq p \), this means that \( |z - p| < r \) and

\[
-\frac{\delta \pi}{2} \leq \text{Arg}(z - p) - \text{Arg}(p - a) \leq \frac{\gamma \pi}{2}.
\]

(3)

Figure 1: The domain \( D \), case \(|\text{Arg}(z - p) - \text{Arg}p| \geq |\text{Arg}(p - a) - \text{Arg}p|\)

If \(|\text{Arg}(z - p) - \text{Arg}p| \geq |\text{Arg}(p - a) - \text{Arg}p|\) (see fig. 1), then, by (??) and (??),

\[
-\frac{\beta \pi}{2} = -\frac{\delta \pi}{2} - \frac{(\beta - \delta)\pi}{2} \leq \text{Arg}(z - p) - \text{Arg}p =
\]

\[
= (\text{Arg}(z - p) - \text{Arg}(p - a)) + (\text{Arg}(p - a) - \text{Arg}p) \leq
\]

\[
\leq \frac{\gamma \pi}{2} + \frac{(\alpha - \gamma)\pi}{2} = \frac{\alpha \pi}{2},
\]

and therefore \( z \in K_+(p, 0, \alpha, \beta, r) \). In the case

\(|\text{Arg}(z - p) - \text{Arg}p| < |\text{Arg}(p - a) - \text{Arg}p|\) (see fig. 2),

we have \( z \in K_+(p, 0, \alpha - \gamma, \beta - \delta, r) \) and therefore \( z \in K_+(p, 0, \alpha, \beta, r) \). Consequently for both cases

\[
K_+(p, a, \gamma, \delta, r) \subset K_+(p, 0, \alpha, \beta, r) \subset \mathbb{C} \setminus D.
\]
Inscribe a disc with center at the origin into the set

\[ K_-(p, 0, \alpha - \gamma, \beta - \delta) \cap D \]

and find its radius \( R(p) \). Denote by \( y \) a point from \( \partial K_-(p, 0, \alpha - \gamma, \beta - \delta) \) such that

\[ |y| = \text{dist}(0, \partial K_-(p, 0, \alpha - \gamma, \beta - \delta)). \]

Then, from the right triangle \( 0, y, p \), we obtain

\[ R(p) = |y| = |p| \sin \frac{\varphi \pi}{2}, \]

where \( \varphi = \min\{\alpha - \gamma, \beta - \delta\} \). Put

\[ R = \min_{p \in \partial D} R(p) = \rho \sin \frac{\varphi \pi}{2}. \]

Then the disc \( \mathbb{B}[0, R] \) is contained in \( K_-(p, 0, \alpha - \gamma, \beta - \delta) \cap D \) for all \( p \in \partial D \). Thus \( \mathbb{B}[0, R] \subset \Omega_D^{\gamma, \delta} \) for all \( D \in A^\alpha_\rho \).

Let us prove that it is impossible to enlarge the constant \( R \) in the last inclusion. For this aim we find a domain \( D_0 \in A^\alpha_\rho \beta \) such that \( \mathbb{B}[0, R] \subset \Omega_{D_0}^{\gamma, \delta} \), but for every \( \varepsilon > 0 \)

\[ \mathbb{B}[0, R + \varepsilon] \not\subset \Omega_{D_0}^{\gamma, \delta}. \]

a) Let us begin with the case \( \alpha, \beta \in (0; 1), \gamma \in [0; \alpha], \delta \in [0; \beta] \). Consider the simply connected domain \( D_0, 0 \in D_0 \), bounded by the logarithmic spirals.
$l_\alpha(\varphi) = \rho e^{i\varphi} e^{\varphi \tan \frac{(1-\alpha)\pi}{2}}, 0 \leq \varphi \leq \frac{\pi}{2},$

$l_\beta(\varphi) = \rho e^{i\varphi} e^{-\varphi \tan \frac{(1-\beta)\pi}{2}}, -\frac{\pi}{2} \leq \varphi \leq 0,$

and the circle

$l(\varphi) = \rho e^{\pi \tan \frac{(1-\alpha)\pi}{2}} e^{i\varphi}, -\pi < \varphi < \pi,$ if $\alpha \geq \beta$

or $l(\varphi) = \rho e^{\pi \tan \frac{(1-\beta)\pi}{2}} e^{i\varphi}, -\pi < \varphi \leq \pi,$ if $\alpha < \beta$ (see fig. 3).

Figure 3: The domain $D_0$, case $\alpha, \beta \in (0; 1)$

We will consider the case $\alpha \geq \beta$ only. The proof for $\alpha < \beta$ is analogous.

Let us check that $D_0 \in A_{\rho}^{\alpha,\beta}$. Note that $\min_{p \in \partial D} |p| = |l_\alpha(0)| = |l_\beta(0)| = \rho$.

Show that domain $D_0$ is $(\alpha, \beta)$–accessible with respect to the origin. Take $p \in \partial D_0$. Prove that the unbounded cone $K_+(p, 0, \alpha, \beta)$ is contained in $\mathbb{C} \setminus D_0$. Denote by $n(p)$ the outward normal to the domain $D_0$ at the point $p$ if such a normal exists. Divide the proof into six cases.

a) 1) Let $p = l_\alpha(\varphi), \varphi \in \left(0; \frac{\pi}{2}\right)$. Then

\[ \Arg p - \Arg n(p) = \Arg p - \left(\Arg l'_\alpha(\varphi) - \frac{\pi}{2}\right) = \]
\[
\varphi - \arg \left( \rho e^{i\varphi} e^{\varphi \tan \left( \frac{1 - \alpha}{2} \varphi \right)} \left( i + \tan \left( \frac{1 - \alpha}{2} \pi \right) \right) \right) + \frac{\pi}{2} =
\]
\[
= \varphi - \varphi - \arctg \frac{1}{\tan \left( \frac{1 - \alpha}{2} \pi \right)} + \frac{\pi}{2} = -\arctg \left( \tan \left( \frac{\alpha \pi}{2} \right) \right) + \frac{\pi}{2} = \frac{(1 - \alpha)\pi}{2}.
\]

By Theorem A, for such \( p \) we have \( K_+(p, 0, \alpha, \beta) \subset \mathbb{C} \setminus D_0 \).

\( a) \) If \( p = l_\beta(\varphi) \), \( \varphi \in (\varphi_0; 0) \), where \( \varphi_0 \) is the solution of the equation 
\[
l(\varphi_0) = l_\beta(\varphi_0).
\]
Then
\[
\arg l'_\beta(\varphi) = \frac{\pi}{2}.
\]

Consequently, by Theorem A, \( K_+(p, 0, \alpha, \beta) \subset \mathbb{C} \setminus D_0 \).

\( a) \) Let \( p = l(\varphi) \), \( \varphi \in (-\pi; \varphi_0) \cup \left( \frac{\pi}{2}; \pi \right) \). In this case
\[
K_+(p, 0, \alpha, \beta) \subset K_+(p, 0, 1, 1) \subset \mathbb{C} \setminus D_0.
\]

\( a) \) Consider
\[
p = l \left( \frac{\pi}{2} \right) = l_\alpha \left( \frac{\pi}{2} \right) = \rho e^{\frac{\pi}{2} \tan \left( \frac{1 - \alpha}{2} \pi \right)}.
\]

Since \( \arg l'_\alpha \left( \frac{\pi}{2} \right) = \frac{\pi}{2} + \frac{\alpha \pi}{2} \), \( \arg l'_\beta \left( \frac{\pi}{2} \right) = \pi \) (here and below in \( a) \)), \( b) \)), \( b) \) we consider one-sided derivatives), and \( \arg \alpha l(\varphi) \), \( \arg l(\varphi) \) are monotone we get
\[
K_+(p, 0, \alpha, \beta) \subset K_+(p, 0, 1, 1) \subset \mathbb{C} \setminus D_0.
\]

\( a) \) Let \( p = l(0) = l_\alpha(0) = \rho \). Since \( \arg l'_\alpha(0) = \frac{\alpha \pi}{2} \) and \( \arg l'_\beta(0) = \pi - \frac{\beta \pi}{2} \), then \( K_+(p, 0, \alpha, \beta) \subset \mathbb{C} \setminus D_0 \) and for all \( \varepsilon_1, \varepsilon_2 \geq 0 \), \( \varepsilon_1^2 + \varepsilon_2^2 \neq 0 \),
\[
K_+(p, 0, \alpha + \varepsilon_1, \beta + \varepsilon_2) \cap D_0 \neq \emptyset.
\]

Moreover, both rays of \( \partial K_+(p, 0, \alpha + \varepsilon_1, \beta + \varepsilon_2) \) intersect the domain \( D_0 \).
The last case is \( p = l(\varphi_0) = l_\beta(\varphi_0) \). Here \( \arg l'(\varphi_0) = \varphi_0 + \frac{\pi}{2} \) and \( \arg l'_\beta(\varphi_0) = \varphi_0 + \pi - \frac{\pi \beta}{2} \). Thus, \( K_+(p, 0, \alpha, \beta) \subset K_+(p, 0, 1, 1) \subset \mathbb{C} \setminus D_0 \).

Consequently \( K_+(p, 0, \alpha, \beta) \subset \mathbb{C} \setminus D_0 \) for all \( p \in \partial D_0 \). Therefore \( D_0 \in A^\alpha,\beta \).

Now we will show that \( \mathbb{B} \left[ 0, \rho \sin \frac{\varphi \pi}{2} \right] \) is the maximal disc, contained in \( \Omega^\gamma,\delta_D \).

Let \( t \in (0, 1 - \varphi) \). Fix
\[
a^* \in \partial \mathbb{B} \left[ 0, \rho \sin \frac{(\varphi + t)\pi}{2} \right] \cap K_-(\rho, 0, \varphi + t, \varphi + t),
\]
such that \( \text{Im} a^* > 0 \) if \( \varphi = \beta - \delta \) and \( \text{Im} a^* < 0 \) if \( \varphi = \alpha - \gamma \) (see fig. 4).

Figure 4: Intersection \( D_0 \) and \( K_+(\rho, a^*, \gamma, \delta) \), case \( \alpha, \beta \in (0; 1) \)

Further we consider the case \( \varphi = \beta - \delta \) only, because the proof for the case \( \varphi = \alpha - \gamma \) is analogous. Let us show that \( a^* \notin \Omega^\gamma,\delta_D \). By definition of \( a^* \),
\[
\text{Arg } \rho - \text{Arg}(\rho - a^*) = (\varphi + t) \frac{\pi}{2}.
\]
(5)
Denote by $l$ the ray, consisting of points $w \in \partial K_+(\rho, a^*, \gamma, \delta)$, $w \neq \rho$, such that
\[
\text{Arg}(w - \rho) - \text{Arg}(\rho - a^*) = -\frac{\delta \pi}{2}.
\]
(6)

By (??) and (??), for every $w \in l$
\[
\text{Arg}(w) - \text{Arg}(\rho) = (\text{Arg}(w) - \text{Arg}(\rho - a^*)) + (\text{Arg}(\rho - a^*) - \text{Arg}(\rho)) = -\frac{\delta \pi}{2} - \frac{(\varphi + t)\pi}{2} = -\frac{(\beta + t)\pi}{2}.
\]
Consequently, $l$ is one of the rays of $\partial K_+(\rho, 0, \alpha, \beta + t)$. As it was proved above (see $a5$)) for every $t > 0$
\[
l \cap D_0 \neq \emptyset.
\]

Therefore $a^* \notin \Omega_{D_0}^{\gamma, \delta}$. Since positive $t$ is arbitrary, we obtain that
\[
\mathbb{B} \left[0, \rho \sin \frac{\varphi \pi}{2}\right]
\]
is the maximal disc, contained in $\Omega_{D_0}^{\gamma, \delta}$.

b) Let $\alpha = 0$, $\beta \in (0; 1)$, $\delta \in [0; \beta]$. Consider the domain $D_0 \subset \mathbb{C}$, $0 \in D_0$, bounded by the logarithmic spiral
\[
l_{\beta}(\varphi) = \rho e^{i \varphi} e^{-\varphi \operatorname{tg} \frac{(1-\beta)\pi}{2}}, \quad -\frac{\pi}{2} \leq \varphi \leq 0,
\]
the circle
\[
l(\varphi) = \rho e^{\frac{\pi}{2} \operatorname{tg} \frac{(1-\beta)\pi}{2}} e^{i \varphi}, \quad \varphi \in \left(-\pi, -\frac{\pi}{2}\right) \cup [0, \pi],
\]
and the segment $\left[\rho; \rho e^{\frac{\pi}{2} \operatorname{tg} \frac{(1-\beta)\pi}{2}}\right]$ (see fig. 5). Show that $D_0 \in A_{\rho}^{0,\beta}$.

Let us prove that domain $D_0$ is $(0, \beta)$–accessible with respect to the origin. Fix $p \in \partial D_0$. Show that $K_+(p, 0, 0, \beta) \subset \mathbb{C}\setminus D_0$.

b1) If $p = l_{\beta}(\varphi)$, $\varphi \in \left(-\frac{\pi}{2}; 0\right)$, then (??) is true (see a1) and
\[
K_+(p, 0, 0, \beta) \subset K_+(p, 0, \alpha, \beta) \subset \mathbb{C}\setminus D_0.
\]

b2) If $p = l(\varphi)$, $\varphi \in \left(-\pi; -\frac{\pi}{2}\right) \cup (0; \pi)$, then
\[
K_+(p, 0, 0, \beta) \subset K_+(p, 0, 1, 1) \subset \mathbb{C}\setminus D_0.
\]
Let \( p = l_\beta(0) = \rho \). In this case \( \arg l'_\beta(0) = \pi - \frac{\beta\pi}{2} \). In addition \( \arg l_\beta(\phi) \) is monotone. Hence, \( K_+(p, 0, 0, \beta) \subset \mathbb{C}\setminus D_0 \).

Consider the case \( p \in \left( \rho; \rho e^{\frac{\pi}{2} \tan \left( \frac{1-\beta}{2} \pi \right)} \right) \). By \( b\beta \), we have

\[
K_+(p, 0, 0, \beta) \subset K_+(p, 0, 0, \beta) \subset \mathbb{C}\setminus D_0.
\]

Let \( p = l \left( -\frac{\pi}{2} \right) = l_\beta \left( -\frac{\pi}{2} \right) = -\rho e^{\frac{\pi}{2} \tan \left( \frac{1-\beta}{2} \pi \right)} \). Since

\[
\arg l'( -\frac{\pi}{2} ) = 0 \quad \text{and} \quad \arg l'_\beta \left( -\frac{\pi}{2} \right) = \frac{(2-\beta)\pi}{2} - \frac{\pi}{2},
\]

and \( \arg l_\beta(\phi) \) is monotone, we get

\[
K_+(p, 0, 0, \beta) \subset K_+(p, 0, 1, 1) \subset \mathbb{C}\setminus D_0.
\]

Summarizing everything proved above we obtain that \( K_+(p, 0, 0, \beta) \subset \mathbb{C}\setminus D_0 \) for all \( p \in \partial D_0 \). Since, in addition, \( \min_{p \in \partial D_0} |p| = |l_\beta(0)| = \rho \), we conclude that \( D_0 \in A^{0,\beta}_\rho \).
Let us check that $B[0,0] = \{0\}$ is the maximal disc, contained in $\Omega_{D_0}^{0,\delta}$.

Suppose that for some $r > 0$

$$B[0, r] \subset \Omega_{D_0}^{0,\delta}.$$ 

Let $z_0 \in B[0, r]$ and $\text{Im} z_0 < 0$. Then, by construction of $D_0$,

$$K_+(\rho, z_0, 0, 0) \cap D_0 \neq \emptyset,$$

see fig. 3. Since $K_+(\rho, z_0, 0, 0) \subset K_+(\rho, z_0, 0, \delta)$, then

$$K_+(\rho, z_0, 0, \delta) \cap D_0 \neq \emptyset.$$ 

Therefore, $z_0 \notin \Omega_{D_0}^{0,\delta}$. This contradiction shows that

$$B[0, r] \notin \Omega_{D_0}^{0,\delta}$$

for every $r > 0$.

c) In the case $\beta = 0$, $\alpha \in (0; 1)$, $\gamma \in [0; \alpha]$, we consider the domain $D_0 \subset \mathbb{C}$, $0 \in D_0$, bounded by the logarithmic spiral

$$l_\alpha(\varphi) = \rho e^{i\varphi} e^{i\varphi^{(1-\alpha)\pi \over 2}}, 0 \leq \varphi \leq {\pi \over 2},$$

the circle

$$l(\varphi) = \rho e^{\pi \varphi^{(1-\alpha)\pi \over 2}} e^{i\varphi}, \quad \varphi \in (-\pi, 0] \cup \left[ {\pi \over 2}, \pi \right],$$

and the segment $\left[ \rho; \rho e^{\pi \varphi^{(1-\alpha)\pi \over 2}} \right]$ (see fig. 6).

Arguing as in case b), taking $z_0$, $\text{Im} z_0 > 0$, we prove that $D_0 \in A^{\alpha,0}_\rho$ and $B[0,0]$ is the maximal disc, contained in $\Omega_{D_0}^{0,0}$.

d) Let $\alpha = \beta = 0$. In this case, the class of $(0,0)$–accessible domains coincides with the class of $0$–accessible domains and the class of starlike with respect to the origin domains (see [11, 12]).

Consider domain $D_0 = \mathbb{C}\setminus l_\rho \in A^{0,0}_\rho$, where $l_\rho = \{\rho t, t \geq 1\}$. Then the set $\Omega_{D_0}^{0,0}$ consists of all points $a \in D_0$ such that $K_+(p, a, 0, 0) \subset l_\rho$ for every $p = \rho \tau$, $\tau \geq 1$. Consequently, $\Omega_{D_0}^{0,0} = \{\rho k, k < 1\}$. Therefore, for all $\varepsilon > 0$

$$B[0, \varepsilon] \notin \Omega_{D_0}^{0,0} \quad \text{and} \quad \bigcap_{D \in A^{0,0}_\rho} \Omega_{D_0}^{0,0} = \{0\}.$$
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References


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