Probl. Anal. Issues Anal. Vol. 12 (30), No 2, 2023, pp. 97–106
DOI: 10.15393/j3.art.2023.12490

UDC 517.54

S. Sümer Eker, B. Şeker, S. Ece

ON NORMALIZED RABOTNOV FUNCTION ASSOCIATED WITH CERTAIN SUBCLASSES OF ANALYTIC FUNCTIONS

Abstract. In this paper, we investigate some sufficient conditions for the normalized Rabotnov function to be in certain subclasses of analytic and univalent functions. The usefulness of the results is depicted by some corollaries and examples.

Key words: *Rabotnov function, univalent, starlike, convex, coefficient bounds and coefficient estimates*

2020 Mathematical Subject Classification: *33E20, 30C45, 30C55*

1. Introduction. The well-known Mittag-Leffler function $E_\alpha(z)$ and its two-parameter version $E_{\alpha,\kappa}(z)$, which are defined, respectively, by (see, for example, [5]):

$$E_\alpha(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(\alpha n + 1)} \quad \text{and} \quad E_{\alpha,\kappa}(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(\alpha n + \kappa)}, \quad (z, \alpha, \kappa \in \mathbb{C}).$$

One can see that this series converges in the whole complex plane for all $\mathrm{Re}(\alpha) > 0$. For all $\mathrm{Re}(\alpha) < 0$, this power series diverges everywhere on $\mathbb{C}\backslash\{0\}$. For $\mathrm{Re}(\alpha) = 0$, the radius of convergence is equal to $\rho = e^{\frac{\pi}{2}|\,\mathrm{Im}\,\alpha|}$. Applications and generalizations of the Mittag-Leffler function have an important place in physics, biology, chemistry, engineering, and other applied sciences.

In 1948, Yu. N. Rabotnov, who worked in solid mechanics, including plasticity, creep theory, hereditary mechanics, failure mechanics, nonelastic stability, composites, and shell theory, introduced a special function applied in viscoelasticity [7]. This function, known today as the Rabotnov fractional exponential function, or briefly the Rabotnov function, is defined as follows:

$$R_{\alpha,\beta}(z) = z^{\alpha} \sum_{n=0}^{\infty} \frac{\beta^n}{\Gamma\left((n+1)(1+\alpha)\right)} z^{n(1+\alpha)}$$

and z^{α} means the principal branch of the corresponding multi-valued function defined in the whole complex plane cut along the negative real semi-axis. The convergence of this series at any values of the argument is evident. Note that for $\alpha = 0$ it reduces to the standard exponential $\exp(\beta z)$. The Rabotnov function is the particular case of the Mittag-Leffler function. The relation between the Rabotnov function and the Mittag-Leffler function can be written as follows:

$$R_{\alpha,\beta}(z) = z^{\alpha} E_{1+\alpha,1+\alpha}(\beta z^{1+\alpha}).$$

For more details, see [5].

A function f is said to be *univalent* in a domain D if it provides a one-to-one mapping onto its image, $f(D)$. Namely, $f\colon D \subseteq \mathbb{C} \to \mathbb{C}$ is said to be *univalent* in D if $f(z_1) = f(z_2)$ for $z_1, z_2 \in D$ implies that $z_1 = z_2$.

If an univalent function f maps D onto a convex domain, then f is called a *convex function*.

A domain D in the plane is said to be *starlike* if the line segment joining any point of D to the origin lies inside D. A function f is called *starlike*, if $f(D)$ is a starlike domain.

Let $\mathcal{A}$ be the class of functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n,$$

which are analytic in the open unit disk $\mathbb{U} = \{z \in \mathbb{C}\colon |z| < 1\}$. We denote by $\mathcal{S}$ the subclass of $\mathcal{A}$ consisting of functions $f \in \mathcal{A}$, which are univalent in $\mathbb{U}$.

It is well-known that a function $f \in \mathcal{S}$ is starlike of order $\gamma (0 \leqslant \gamma < 1)$ if and only if

$$\operatorname{Re}\left\{\frac{zf'(z)}{f(z)}\right\} > \gamma, \qquad (z \in \mathbb{U}).$$

Denote by $\mathcal{S}^*(\gamma)$ the class of all functions that are starlike of order γ. Furthermore, a function $f \in \mathcal{S}$ is convex of order $\gamma\,(0 \leqslant \gamma < 1)$ if and only if

$$\operatorname{Re}\left\{1 + \frac{zf''(z)}{f'(z)}\right\} > \gamma, \qquad (z \in \mathbb{U}).$$

Denote the class of convex functions of order γ by $\mathcal{C}(\gamma)$. Note that $\mathcal{C}(0)=\mathcal{C}$, and $\mathcal{S}^*(0)=\mathcal{S}^*$, where $\mathcal{C}$ and $\mathcal{S}^*$ denote the classes of convex and starlike functions, respectively. Furthermore, for all $0\leqslant\gamma<1$, we have $\mathcal{C}(\gamma)\subset\mathcal{S}^*(\gamma)$. For more details about the univalent functions, see [3].

For $0\leqslant\gamma<1$ and $0\leqslant\lambda<1$, Thulasiram et al. [11] introduced the subclass $\mathcal{G}(\lambda,\gamma)$ of functions $f\in\mathcal{A}$ that satisfy the condition:

$$\operatorname{Re}\left(\frac{zf'(z)+\lambda z^2 f''(z)}{f(z)}\right)>\gamma, \qquad (z\in\mathbb{U}).$$

Furthermore, let us define $f\in\mathcal{K}(\lambda,\gamma)\Leftrightarrow zf'\in\mathcal{G}(\lambda,\gamma)$. Clearly, for $\lambda=0$ we have $\mathcal{G}(0,\gamma)=\mathcal{S}^*(\gamma)$ and $\mathcal{K}(0,\gamma)=\mathcal{C}(\gamma)$.

In order to obtain our results, we need the following Lemmas given by Thulasiram et al and Şeker et al.

Lemma 1. *[11] A function $f\in\mathcal{A}$ belongs to the class $\mathcal{G}(\lambda,\gamma)$ if*

$$\sum_{n=2}^{\infty}(n+\lambda n(n-1)-\gamma)\,|a_n|\leqslant 1-\gamma.$$

Lemma 2. *[8] A function $f\in\mathcal{A}$ belongs to the class $\mathcal{K}(\lambda,\gamma)$ if*

$$\sum_{n=2}^{\infty}n\,(n+\lambda n(n-1)-\gamma)\,|a_n|\leqslant 1-\gamma.$$

Starlikeness, convexity, close-to-convexity, and some other geometric properties of special functions, such as Bessel, Struve, Wright, Dini, Mittag-Leffler, Miller-Ross, hypergeometric, etc., have been studied by many mathematicians recently (see, for example, [8], [4], [9], [10], [1], [6], [2]). Motivated by the these works, we obtained sufficient conditions for the Rabotnov function to be in the classes $\mathcal{G}(\beta,\alpha)$ and $\mathcal{K}(\beta,\alpha)$.

Throughout this paper, we restrict our attention to the case of real-valued $\alpha\geqslant 0$, $\beta>0$, and $z\in\mathbb{U}$. It is clear that the Rabotnov function $R_{\alpha,\beta}(z)$ does not belong to the family $\mathcal{A}$. Thus, it is natural to consider the following normalization of the Rabotnov functions:

$$\mathbb{R}_{\alpha,\beta}(z)=z^{1/(1+\alpha)}\Gamma(1+\alpha)R_{\alpha,\beta}(z^{1/(1+\alpha)})=z+\sum_{n=2}^{\infty}\frac{\beta^{n-1}\Gamma(1+\alpha)}{\Gamma((1+\alpha)n)}z^n. \quad (1)$$

The following lemma given by Sümer Eker et al. allows us to prove our theorems:

Lemma 3. *[9] If $n \in \mathbb{N}$ and $\alpha \geqslant 0$, then*

$$(1+\alpha)^{n-1}(n-1)!\Gamma(1+\alpha) \leqslant \Gamma\left((1+\alpha)n\right).$$

Proof. Let us prove the lemma by induction for all $n \in \mathbb{N} = \{1, 2, \ldots\}$. The case $n = 1$ is trivial. Now assume that the inequality holds for $n = k$. Hence, by the induction assumption, we get

$$\begin{aligned}(1+\alpha)^k k!\Gamma(1+\alpha) &= (1+\alpha)k(1+\alpha)^{k-1}(k-1)!\Gamma(1+\alpha) \leqslant \\ &\leqslant (1+\alpha)k\Gamma((1+\alpha)k) = \Gamma((1+\alpha)k+1) \leqslant \\ &\leqslant \Gamma((1+\alpha)(k+1)).\end{aligned}$$

This completes the proof. □

From Lemma 3, for $n \in \mathbb{N}$ and $\alpha \geqslant 0$, we can write:

$$\frac{\Gamma(1+\alpha)}{\Gamma\left((1+\alpha)n\right)} \leqslant \frac{1}{(1+\alpha)^{n-1}(n-1)!}. \tag{2}$$

2. Main Results.

Theorem 1. *Let $\alpha \geqslant 0$ and $\beta > 0$. If the following condition is satisfied:*

$$e^{\frac{\beta}{1+\alpha}}\left(\frac{\lambda\beta^2}{(1+\alpha)^2} + \frac{(1+2\lambda)\beta}{1+\alpha} + 1 - \gamma\right) \leqslant 2(1-\gamma),$$

then the normalized Rabotnov function $\mathbb{R}_{\alpha,\beta}(z)$ given by (1) belongs to the class $\mathcal{G}\,(\lambda,\gamma)$.

Proof. By virtue of Lemma 1, it suffices to show that

$$\sum_{n=2}^{\infty}\left(n + \lambda n\,(n-1) - \gamma\right)\left|\frac{\beta^{n-1}\Gamma(1+\alpha)}{\Gamma((1+\alpha)n)}\right| \leqslant 1 - \gamma. \tag{3}$$

Let

$$L_1(\lambda,\gamma,\alpha) = \sum_{n=2}^{\infty}\left(n + \lambda n\,(n-1) - \gamma\right)\frac{\beta^{n-1}\Gamma(1+\alpha)}{\Gamma((1+\alpha)n)}.$$

Writing

$$\begin{cases} n^3 = (n-1)(n-2)(n-3) + 6(n-1)(n-2) + 7(n-1) + 1, \\ n^2 = (n-1)(n-2) + 3(n-1) + 1, \\ n = (n-1) + 1, \end{cases}$$

and using (2), we get

$$
\begin{aligned}
L_1(\lambda,\gamma,\alpha) &= \sum_{n=2}^{\infty}\left(\lambda n^2+(1-\lambda)n-\gamma\right)\frac{\beta^{n-1}\Gamma(1+\alpha)}{\Gamma((1+\alpha)n)} = \\
&= \sum_{n=2}^{\infty}\left[\lambda(n-1)(n-2)+(1+2\lambda)\,(n-1)+1-\gamma\right]\frac{\beta^{n-1}\Gamma(1+\alpha)}{\Gamma((1+\alpha)n)} = \\
&= \sum_{n=2}^{\infty}\frac{\lambda(n-1)(n-2)\beta^{n-1}\Gamma(1+\alpha)}{\Gamma((1+\alpha)n)}+\sum_{n=2}^{\infty}\frac{(1+2\lambda)(n-1)\beta^{n-1}\Gamma(1+\alpha)}{\Gamma((1+\alpha)n)}+ \\
&\quad +\sum_{n=2}^{\infty}\frac{(1-\gamma)\beta^{n-1}\Gamma(1+\alpha)}{\Gamma((1+\alpha)n)} \leqslant \\
&\leqslant \lambda\sum_{n=2}^{\infty}\frac{(n-1)(n-2)\beta^{n-1}}{(1+\alpha)^{n-1}(n-1)!}+(1+2\lambda)\sum_{n=2}^{\infty}\frac{(n-1)\beta^{n-1}}{(1+\alpha)^{n-1}(n-1)!}+ \\
&\quad +(1-\gamma)\sum_{n=2}^{\infty}\frac{\beta^{n-1}}{(1+\alpha)^{n-1}(n-1)!} = \\
&= \lambda\sum_{n=3}^{\infty}\frac{\beta^{n-1}}{(1+\alpha)^{n-1}(n-3)!}+(1+2\lambda)\sum_{n=2}^{\infty}\frac{\beta^{n-1}}{(1+\alpha)^{n-1}(n-2)!}+ \\
&\quad +(1-\gamma)\sum_{n=2}^{\infty}\frac{\beta^{n-1}}{(1+\alpha)^{n-1}(n-1)!} = \\
&= \frac{\lambda\beta^2}{(1+\alpha)^2}e^{\frac{\beta}{1+\alpha}}+\frac{(1+2\lambda)\beta}{(1+\alpha)}e^{\frac{\beta}{1+\alpha}}+(1-\gamma)\left(e^{\frac{\beta}{1+\alpha}}-1\right).
\end{aligned}
$$

Thus (3) holds if the following condition is satisfied:

$$
\frac{\lambda\beta^2}{(1+\alpha)^2}e^{\frac{\beta}{1+\alpha}}+\frac{(1+2\lambda)\beta}{(1+\alpha)}e^{\frac{\beta}{1+\alpha}}+(1-\gamma)\left(e^{\frac{\beta}{1+\alpha}}-1\right)\leqslant 1-\gamma,
$$

which is equivalent to

$$
(1-\gamma)\left(2-e^{\frac{\beta}{1+\alpha}}\right)-\frac{\lambda\beta^2}{(1+\alpha)^2}e^{\frac{\beta}{1+\alpha}}-\frac{(1+2\lambda)\beta}{1+\alpha}e^{\frac{\beta}{1+\alpha}}\geqslant 0
$$

or, equivalently,

$$
e^{\frac{\beta}{1+\alpha}}\left(\frac{\lambda\beta^2}{(1+\alpha)^2}+\frac{(1+2\lambda)\beta}{1+\alpha}+1-\gamma\right)\leqslant 2(1-\gamma).
$$

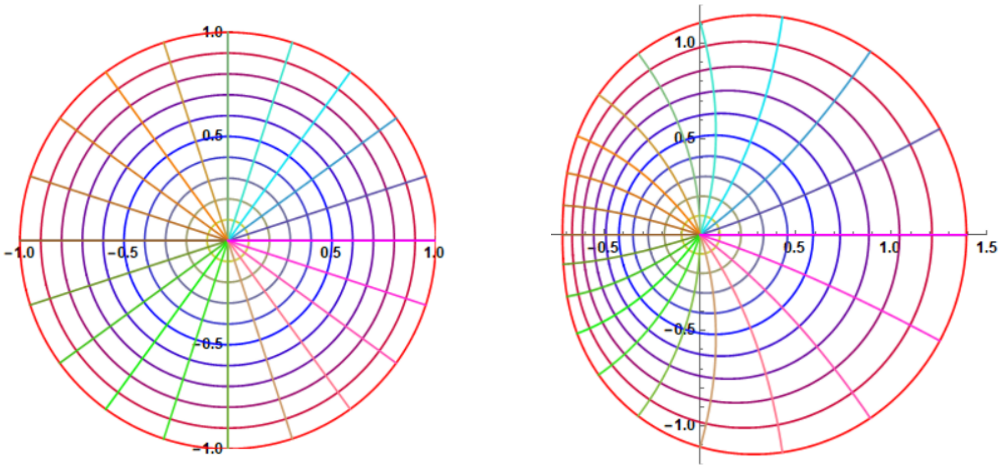


Figure 1: Mapping of $\mathbb{R}_{0,\frac{32}{100}}(z)$ over $\mathbb{U}$

This completes the proof of Theorem 1. □

Example. The function $\mathbb{R}_{0,\frac{32}{100}}(z)$ is in the class $\mathcal{G}(\lambda,\gamma)$. Figure 1 shows the case of $\lambda = \gamma = 1/10$.

Theorem 2. *Let $\alpha \geqslant 0$ and $\beta > 0$. If the following condition is satisfied:*

$$e^{\frac{\beta}{1+\alpha}}\left(\frac{\lambda\beta^3}{(1+\alpha)^3}+\frac{(1+5\lambda)\beta^2}{(1+\alpha)^2}+\frac{(3+4\lambda-\gamma)\beta}{1+\alpha}+1-\gamma\right)\leqslant 2(1-\gamma),$$

then the normalized Rabotnov function $\mathbb{R}_{\alpha,\beta}(z)$ given by (1) belongs to the class $\mathcal{K}(\lambda,\gamma)$.

Proof. By virtue of Lemma 2, it suffices to show that

$$\sum_{n=2}^{\infty} n\,(n+\lambda n\,(n-1)-\gamma)\left|\frac{\beta^{n-1}\Gamma(1+\alpha)}{\Gamma((1+\alpha)n)}\right| \leqslant 1-\gamma. \tag{4}$$

Let

$$L_2(\lambda,\gamma,\alpha) = \sum_{n=2}^{\infty} n\,(n+\lambda n\,(n-1)-\gamma)\,\frac{\beta^{n-1}\Gamma(1+\alpha)}{\Gamma((1+\alpha)n)}.$$

Following the expressions for n^3, n^2, and n, as in the proof of Theorem 1, and using (2), we obtain

$$L_2(\lambda,\gamma,\alpha) = \sum_{n=2}^{\infty}\left(\lambda n^3+(1-\lambda)n^2-\gamma n\right)\frac{\beta^{n-1}\Gamma(1+\alpha)}{\Gamma((1+\alpha)n)} =$$

$$= \sum_{n=2}^{\infty} \Big[\lambda(n-1)(n-2)(n-3) + (1+5\lambda)(n-2)\,(n-1) +$$

$$+ (3+4\lambda-\gamma)(n-1) + (1-\gamma) \Big] \frac{\beta^{n-1}\Gamma(1+\alpha)}{\Gamma((1+\alpha)n)} =$$

$$= \sum_{n=2}^{\infty} \frac{\lambda(n-1)(n-2)(n-3)\beta^{n-1}\Gamma(1+\alpha)}{\Gamma((1+\alpha)n)} +$$

$$+ \sum_{n=2}^{\infty} \frac{(1+5\lambda)(n-1)(n-2)\beta^{n-1}\Gamma(1+\alpha)}{\Gamma((1+\alpha)n)} +$$

$$+ \sum_{n=2}^{\infty} \frac{(3+4\lambda-\gamma)(n-1)\beta^{n-1}\Gamma(1+\alpha)}{\Gamma((1+\alpha)n)} + \sum_{n=2}^{\infty} \frac{(1-\gamma)\beta^{n-1}\Gamma(1+\alpha)}{\Gamma((1+\alpha)n)} \leqslant$$

$$\leqslant \lambda \sum_{n=2}^{\infty} \frac{(n-1)(n-2)(n-3)\beta^{n-1}}{(1+\alpha)^{n-1}(n-1)!} + (1+5\lambda) \sum_{n=2}^{\infty} \frac{(n-1)(n-2)\beta^{n-1}}{(1+\alpha)^{n-1}(n-1)!} +$$

$$+ (3+4\lambda-\gamma) \sum_{n=2}^{\infty} \frac{(n-1)\beta^{n-1}}{(1+\alpha)^{n-1}(n-1)!} + (1-\gamma) \sum_{n=2}^{\infty} \frac{\beta^{n-1}}{(1+\alpha)^{n-1}(n-1)!} =$$

$$= \lambda \sum_{n=4}^{\infty} \frac{\beta^{n-1}}{(1+\alpha)^{n-1}(n-4)!} + (1+5\lambda) \sum_{n=3}^{\infty} \frac{\beta^{n-1}}{(1+\alpha)^{n-1}(n-3)!} +$$

$$+ (3+4\lambda-\gamma) \sum_{n=2}^{\infty} \frac{\beta^{n-1}}{(1+\alpha)^{n-1}(n-2)!} + (1-\gamma) \sum_{n=2}^{\infty} \frac{\beta^{n-1}}{(1+\alpha)^{n-1}(n-1)!} =$$

$$= \frac{\lambda\beta^3}{(1+\alpha)^3} e^{\frac{\beta}{1+\alpha}} + \frac{(1+5\lambda)\beta^2}{(1+\alpha)^2} e^{\frac{\beta}{1+\alpha}} + \frac{(3+4\lambda-\gamma)\beta}{(1+\alpha)} e^{\frac{\beta}{1+\alpha}} + (1-\gamma)(e^{\frac{\beta}{1+\alpha}} - 1).$$

Thus, (4) holds if the following condition is satisfied:

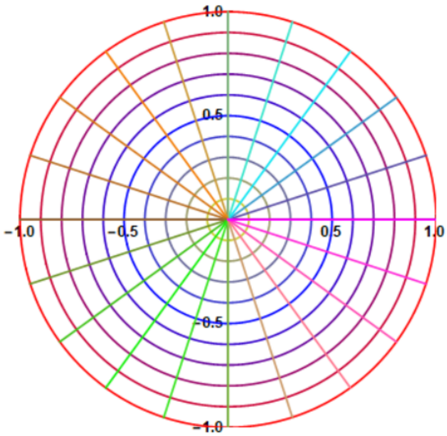
$$\frac{\lambda\beta^3}{(1+\alpha)^3} e^{\frac{\beta}{1+\alpha}} + \frac{(1+5\lambda)\beta^2}{(1+\alpha)^2} e^{\frac{\beta}{1+\alpha}} +$$

$$+ \frac{(3+4\lambda-\gamma)\beta}{(1+\alpha)} e^{\frac{\beta}{1+\alpha}} + (1-\gamma)\left(e^{\frac{\beta}{1+\alpha}} - 1\right) \leqslant 1-\gamma,$$

which is equivalent to

$$e^{\frac{\beta}{1+\alpha}} \left(\frac{\lambda\beta^3}{(1+\alpha)^3} + \frac{(1+5\lambda)\beta^2}{(1+\alpha)^2} + \frac{(3+4\lambda-\gamma)\beta}{1+\alpha} + 1 - \gamma \right) \leqslant 2(1-\gamma).$$

This, evidently, completes the proof of the Theorem. □



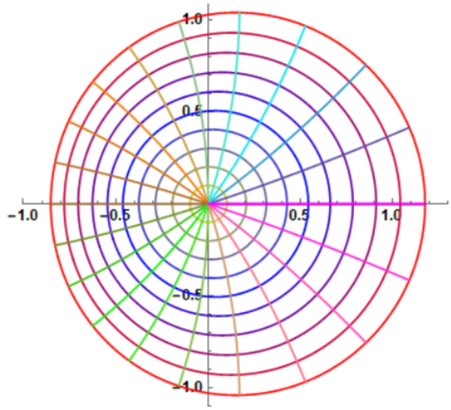


Figure 2: Mapping of $\mathbb{R}_{0,\frac{9}{100}}(z)$ over $\mathbb{U}$

Example. The function $\mathbb{R}_{0,\frac{9}{100}}(z)$ is in the class $\mathcal{K}(\lambda,\gamma)$. Figure 2 shows the case of $\lambda = 1/20$ and $\gamma = 1/10$.

By setting $\lambda = 0$ in our theorems, we obtain the following corollaries:

Corollary 1. *Let $\alpha \geqslant 0$ and $\beta > 0$. If the following condition is satisfied:*

$$e^{\frac{\beta}{1+\alpha}}\left(\frac{\beta}{1+\alpha} + 1 - \gamma\right) \leqslant 2(1-\gamma),$$

then the normalized Rabotnov function $\mathbb{R}_{\alpha,\beta}(z)$ given by (1) belongs to the class $\mathcal{S}^*(\gamma)$.

Corollary 2. *Let $\alpha \geqslant 0$ and $\beta > 0$. If the following condition is satisfied:*

$$e^{\frac{\beta}{1+\alpha}}\left(\frac{\beta^2}{(1+\alpha)^2} + \frac{(3-\gamma)\beta}{1+\alpha} + 1 - \gamma\right) \leqslant 2(1-\gamma),$$

then the normalized Rabotnov function $\mathbb{R}_{\alpha,\beta}(z)$ given by (1) belongs to the class $\mathcal{C}(\gamma)$.

If we take $\lambda = 0$ and $\gamma = 0$ in our theorems, the results of the calculations for obtained inequalities coincide with the theorems given by Eker and Ece [9].

Corollary 3. *Let $\alpha \geqslant 0$ and $\beta > 0$. If $\alpha > \frac{\beta}{W(2e)-1} - 1$, where W is the Lambert W function, then the normalized Rabotnov function $\mathbb{R}_{\alpha,\beta}(z)$ is starlike in $\mathbb{U}$.*

Corollary 4. *Let* $\alpha \geqslant 0$ *and* $\beta > 0$. *If* $\frac{\beta}{1+\alpha} < 0.199496$, *then the normalized Rabotnov function* $\mathbb{R}_{\alpha,\beta}(z)$ *is convex in* $\mathbb{U}$.

References

[1] Bansal D., Prajapat J. K. *Certain geometric properties of the Mittag-Leffler functions.* Complex Var. Elliptic Equ., 2016, vol. 61(3), pp 338–350.
DOI: https://doi.org/10.1080/17476933.2015.1079628

[2] Baricz A. *Geometric properties of generalized Bessel functions.* Publ Math Debrecen., 2008, vol. 73, pp 155–178.

[3] Duren P. L. *Univalent Functions.* Grundlehren der Mathematischen Wissenschaften. New York, NY, USA: Springer-Verlag, 1983.

[4] Ece S., Sümer Eker S. *Characterizations for certain subclasses of starlike and convex functions associated with generalized Dini functions.* Afr. Mat., 2022, vol. 33, pp.10. DOI: https://doi.org/10.1007/s13370-021-00955-w

[5] Gorenflo R., Kilbas A. A., Mainardi F., Rogosin S. V. *Mittag-Leffler Functions, Related Topics and Applications.* Springer-Verlag Berlin Heidelberg, 2014.

[6] Prajapat J. K. *Certain geometric properties of the Wright functions.* Integral Transforms Spec. Funct., 2015, vol. 26 (3), pp. 203–212.
DOI: https://doi.org/10.1080/10652469.2014.983502

[7] Rabotnov Y. *Equilibrium of an Elastic Medium with After-Effect.* Prikladnaya Matematika i Mekhanika, 12, 1948, 1, pp. 53-62 (in Russian), Reprinted: Fract. Calc. and Appl. Anal., 2014, 17, pp. 684-696.
DOI: https://doi.org/10.2478/s13540-014-0193-1

[8] Şeker B., Sümer Eker S. *On Certain Subclasses of Starlike and Convex Functions Associated with Pascal Distribution Series.* Turk J Math., 2020, vol. 44, pp. 1982-1989. DOI: https://doi.org/10.3906/mat-2003-12

[9] Sümer Eker S., Ece S. *Geometric Properties Of Normalized Rabotnov Function.* Hacet. J. Math. Stat., 2022, vol. 51 (5), pp. 1248-1259.
DOI: https://doi.org/10.15672/hujms.980307

[10] Sümer Eker S., Ece S. *Geometric Properties of the Miller-Ross Functions.* Iran J Sci Technol Trans Sci, 2022, vol. 46 (2), pp 631-636.
DOI: https://doi.org/10.1007/s40995-022-01268-8

[11] Thulasiram T., Suchithra K., Sudharsan T., Murugusundaramoorthy G. *Some inclusion results associated with certain subclass of analytic functions inolving Hohlov operator.* Rev. R. Acad. Cienc. Exactas Fis. Nat. Ser. A Math. RACSAM, 2014, vol. 108(2), pp. 711-720.
DOI: https://doi.org/10.1007/s13398-013-0135-5

Received October 27, 2022.
In revised form, February 25, 2023.
Accepted February 26, 2023.
Published online March 11, 2023.

Dicle University
Department of Mathematics,
Faculty of Science, Diyarbakır, Türkiye

Sevtap Sümer Eker
E-mail: sevtaps@dicle.edu.tr

Bilal Şeker
E-mail: bilalseker1980@gmail.com

Sadettin Ece
E-mail: ecesadettin47@gmail.com