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FIXED-POINT THEOREMS USING INTERPOLATIVE BOYD-WONG-TYPE CONTRACTIONS AND INTERPOLATIVE MATKOWSKI-TYPE CONTRACTIONS ON PARTIAL B-METRIC SPACES

Abstract. This article introduces interpolative contractions of both Boyd-Wong and Matkowski types in the framework of partial b-metric spaces. We derive fixed-point theorems for these two contractions and incorporate examples to emphasize the practical relevance of our findings.

Key words: *partial b-metric space, interpolative Boyd-Wong contraction, interpolative Matkowski contraction, fixed point*

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1. Introduction. The concept of fixed point is a fundamental idea in mathematics with broad applications in various fields, including computer science, physics, and economics. A fixed point of a function is a point that remains unchanged when the function is applied to it. The study of fixed point theory aims at establishing conditions under which functions possess fixed points and characterizing the properties of these fixed points. One of the most well-known results in the fixed point theory is the Banach Contraction Principle [4], which states that if a function is a contraction mapping on a complete metric space, then it has a unique fixed point. This principle has numerous applications in areas such as differential equations, functional analysis, and optimization. The basic ideas of the fixed point theory have been extended and generalized in various ways to accommodate more complex mathematical structures and applications. For example, fixed-point theorems have been developed for more general metric spaces, such as partially ordered sets and cone metric spaces.

The concept of partial b-metric space [17] is a generalization of both partial metric space [14] and b-metric space [6], providing a more flexible framework for analyzing various mathematical structures. Partial b-metric spaces combine the properties of partial metrics, which allow the distance of a point from itself to be non-zero, with the properties of b-metric with a relaxation in the triangular inequality. Researchers have proved various fixed-point theorems for different types of contraction mappings in the setting of partial b-metric spaces. These results extend and generalize the well-known Banach contraction principle from metric spaces to partial b-metric spaces [17].

Interpolative contractions are a refinement of well-known contraction mappings, such as Kannan, Reich, and Ciric-Reich-Rus contractions, which have been extensively studied in the fixed point theory. These interpolative contractions incorporate an interpolative mechanism that allows for greater flexibility and generalization of the contraction conditions leading to the existence of fixed points even when the traditional contraction conditions are not satisfied. In 2018, Karapinar introduced interpolative Kannan-type contraction [12], which gave a new research direction in the fixed-point theory by the interpolative approach. Interpolative contractions have been investigated in various metric spaces, including b-metric spaces [6], partial b-metric spaces [17], and uniform spaces, etc. For further literature, one can refer to [2], [3], [8], [10], [11], [13], [16], [18], [19].

We begin with the definition of the b-metric space by Czerwik [6]:

Definition 1. [6] *Let X be a nonempty set. A map $d\colon X \times X \to [0,\infty)$ defines a b-metric on X if for all $x, y, z \in X$, there exists $s \geqslant 1$, such that d satisfies the following properties:*

1) $d(x,y) = 0$ *if and only if* $x = y$*;*
2) $d(x,y) = d(y,x)$*;*
3) $d(x,y) \leqslant s[d(x,z) + d(z,y)]$.

Then (X,d,s) is called a b-metric space with coefficient s.

Consequently, S. Matthews in 1994 introduced the following concept of partial metric space [14]:

Definition 2. [14] *Let X be a nonempty set. A map $P\colon X \times X \to [0,\infty)$ defines a partial metric on X if for all $x, y, z \in X$, P satisfies the following properties:*

1) $P(x,y) = P(x,x) = P(y,y)$ *if and only if* $x = y$*;*
2) $P(x,y) = P(y,x)$*;*

3) $P(x,y) \leqslant P(x,z) + P(z,y) - P(z,z)$.

Then (X,P) *is called a partial metric space.*

Satish Shukla [17] in 2014 gave the definition of partial b-metric space:

Definition 3. [17] *Let* X *be a nonempty set. A map* $P\colon X \times X \to [0,\infty)$ *defines a partial b-metric on* X *if for all* $x,y,z \in X$*, there exists* $s \geqslant 1$*, such that* P *satisfies the following properties:*

1) $P(x,y) = P(x,x) = P(y,y)$ *if and only if* $x = y$;
2) $P(x,y) = P(y,x)$;
3) $P(x,y) \leqslant s[P(x,z) + P(z,y)] - P(z,z)$.

Then (X,P,s) *is called a partial b-metric space with coefficient* s.

Definition 4. [17] *Let* (X,P,s) *be a partial b-metric space. Then*

1) *A sequence* (w_k) *in* X *is said to converge to* w *if* $\lim_{k\to\infty} P(w_k,w) = P(w,w)$.
2) *A sequence* (w_k) *in* X *is said to be Cauchy in* X *if* $\lim_{k,l\to\infty} P(w_k,w_l)$ *exists.*
3) *A partial b-metric space is called complete if for each Cauchy sequence* (w_k) *in* X*, there is* w *in* X*, such that* $\lim_{k,l\to\infty} P(w_k,w_l) = \lim_{k\to\infty} P(w_k,w) = P(w,w)$.

In 2020, Aydi et al. [1] gave the definitions of interpolative Matkowski-type contraction and interpolative Boyd-Wong-type contraction in metric spaces by approaching the Boyd-Wong and Matkowski contractions [5] via interpolative method.

We shall use the following notations subsequently.

$\mathcal{R} := \{\tau \mid \tau\colon [0,\infty) \to [0,\infty),\ \tau(0) = 0,\ \tau(t) < t,$ for $t > 0, \tau$ is upper semi-continuous$\}$,

$\mathbb{N} :=$ the set of natural numbers,

$\mathcal{G} := \{\mathcal{J} \mid \mathcal{J}\colon [0,\infty) \to [0,\infty),$ non-decreasing, $\lim_{k\to\infty} \mathcal{J}^k(v) = 0$ for $v > 0\}$,

$Fix(S) := \{x \mid S\colon X \to X$ such that $S(x) = x\}$.

Matkowski [15] gave a simple but valuable result that is instrumental in proving the subsequent results:

Lemma 1. [15] *Let* $\mathcal{J} \in \mathcal{G}$*. Then* $\mathcal{J}(v) < v$ *for each* $v > 0$ *and* $\mathcal{J}(0) = 0$.

Lemma 2. [7] *Let (X, P, s) be a partial b-metric space with coefficient $s \geqslant 1$ and let (x_n) be a sequence in X, such that $(P(x_n, x_{n+1}))$ is non-increasing and $\lim_{n\to\infty} P(x_n, x_{n+1}) = 0$. If (x_{2n}) is not a Cauchy sequence, then there exist $k > 0$ and two sequences (m_k) and (n_k) of positive integers, such that $m_k > n_k > k$ and the following four sequences $(P(x_{2m_k}, x_{2n_k}))$, $(P(x_{2m_k}, x_{2n_k+1}))$, $(P(x_{2m_k-1}, x_{2n_k}))$, $(P(x_{2m_k-1}, x_{2n_k+1}))$ satisfy*

$$\varepsilon \leqslant \liminf_{k\to\infty} P(x_{2m_k}, x_{2n_k}) \leqslant \limsup_{k\to\infty} P(x_{2m_k}, x_{2n_k}) \leqslant s\varepsilon,$$
$$\frac{\varepsilon}{s} \leqslant \liminf_{k\to\infty} P(x_{2m_k}, x_{2n_k+1}) \leqslant \limsup_{k\to\infty} P(x_{2m_k}, x_{2n_k+1}) \leqslant s^2\varepsilon,$$
$$\varepsilon \leqslant \liminf_{k\to\infty} P(x_{2m_k-1}, x_{2n_k}) \leqslant \limsup_{k\to\infty} P(x_{2m_k-1}, x_{2n_k}) \leqslant s\varepsilon,$$
$$\frac{\varepsilon}{s^2} \leqslant \liminf_{k\to\infty} P(x_{2m_k-1}, x_{2n_k+1}) \leqslant \limsup_{k\to\infty} P(x_{2m_k-1}, x_{2n_k+1}) \leqslant s^2\varepsilon.$$

Definition 5. [5] *Let (X, d) be a metric space. An operator T on X defines a Boyd-Wong contraction if for all x,$y \in X$, there exists $\tau \in \mathcal{R}$, such that T satisfies the following condition:*

$$d(Tx, Ty) \leqslant \tau(d(x, y)).$$

Definition 6. [5] *Let (X, d) be a metric space. An operator T on X defines a Matkowski contraction if for all $x, y \in X$ there exists $\mathcal{J} \in \mathcal{G}$, such that T satisfies the following condition:*

$$d(Tx, Ty) \leqslant \mathcal{J}(d(x, y)).$$

Definition 7. [1] *Let (X, d) be a metric space. Then the operator $T\colon X \to X$ is called an interpolative Boyd-Wong type contraction in metric space, if there are $p, q, r \in (0, 1)$, such that $\lambda = p + q + r < 1$ and a monotonically increasing function $\tau \in \mathcal{R}$ with*

$$d(Tx, Ty) \leqslant \tau([d(x, y)]^p[d(x, Tx]^q[d(y, Ty)]^r[\frac{1}{2}(d(Tx, y)+d(x, Ty)]^{1-\lambda}),$$
$$\text{for each } x, y \in X\backslash Fix(T).$$

Theorem 1. [1] *Let (X, d) be a complete metric space and the operator T on X be an interpolative Boyd-Wong type contraction. Then T has a fixed point in X.*

Definition 8. [1] *Let (X, d) be a metric space. Then the operator $T: X \to X$ is called an interpolative Matkowski type contraction in metric space, if there are $p, q, r \in (0, 1)$, such that $\lambda = p + q + r < 1$ and $\mathcal{J} \in \mathcal{G}$ with*

$$d(Tx, Ty) \leqslant \mathcal{J}([d(x, y)]^p [d(x, Tx)]^q [d(y, Ty)]^r [\frac{1}{2}(d(Tx, y) + d(x, Ty)]^{1-\lambda}),$$
$$\text{for each } x, y \in X \backslash Fix(T).$$

Theorem 2. [1] *Let (X, d) be a complete metric space and T be an interpolative Matkowski-type contraction. Then T has a fixed point in X.*

This paper introduces interpolative Boyd-Wong and Matkowski-type contractions in partial b-metric spaces, inspired by the work done by Aydi et al. [1] on interpolative Boyd-Wong contractions in metric spaces. The authors prove fixed-point theorems for these new contractions and provide examples in support of the proved results.

2. Interpolative Boyd-Wong type contraction and fixed point theorem. Aydi et al. [1] introduced interpolative Boyd-Wong type contraction in a metric space. In this section, we give the definition of interpolative Boyd-Wong contraction in the setting of partial b-metric space and proved the fixed point results. We begin this section with the following definitions:

Definition 9. *Let (X, P, s) be a partial b-metric space with coefficient $s \geqslant 1$. Then the operator T from X to X is called an interpolative Boyd-Wong type contraction in partial b-metric space, if there are $\alpha, \beta, \gamma \in (0, 1)$, such that $\lambda = \alpha + \beta + \gamma < 1$ and a monotonically increasing function $\phi \in \mathcal{R}$ with*

$$P(Tx, Ty) \leqslant \phi\Big([P(x, y)]^\alpha [P(x, Tx)]^\beta [P(y, Ty)]^\gamma$$
$$[\frac{1}{2}(P(Tx), y) + P(y, Ty) - P(y, y))]^{1-\lambda}\Big)$$
$$\text{for all } x, y \in X \backslash Fix(T). \quad (1)$$

Remark. *The expression in the right-hand side of the inequality (1) is non-negative and not symmetric in x and y.*

Theorem 3. *Let (X, P, s) be a complete partial b-metric space with coefficient $s \geqslant 1$ and T be an interpolative Boyd-Wong type contraction. Then T has a unique fixed point x (say) in X and $P(x, x) = 0$.*

Proof. For $x_0 \in X$, define $T^k x_0 = x_k$ for each $k \in \mathbb{N} \cup \{0\}$. If $x_k = x_{k+1}$ for some $k \in \mathbb{N}$, then x_k is fixed point of T. Assume that $x_k \neq x_{k+1}$, for all $k \in \mathbb{N} \cup \{0\}$. Let $x = x_{k-1}$ and $y = x_k$ in (1); then

$$P(x_k, x_{k+1}) = P(Tx_{k-1}, Tx_k) \leqslant \phi\Big((P(x_{k-1}, x_k))^\alpha (P(x_{k-1}, x_k))^\beta$$
$$(P(x_k, x_{k+1}))^\gamma \big(\frac{1}{2}(P(x_k, x_k) + P(x_k, x_{k+1}) - P(x_k, x_k))\big)^{1-\lambda}\Big),$$

where $\alpha, \beta, \gamma \in (0,1)$ and $\lambda = \alpha + \beta + \gamma < 1$.

Let $d_k = P(x_k, x_{k+1})$. Then we have

$$d_k \leqslant \phi\Big((d_{k-1})^{\alpha+\beta}(d_k)^\gamma (\frac{1}{2}(d_k))^{1-\lambda}\Big) \leqslant \phi\Big((d_{k-1})^{\alpha+\beta}(d_k)^{\gamma+1-\lambda}\Big). \tag{2}$$

Suppose $d_k > d_{k-1}$ for some $k \in \mathbb{N}$. In view of the above inequality, we have $d_k \leqslant \phi\left((d_{k-1})^{\alpha+\beta}(d_k)^{\gamma+1-\lambda}\right) \leqslant \phi(d_k)$, which contradicts the fact that $\phi(t) < t$ for all $t > 0$. Thus, $d_k \leqslant d_{k-1}$ for all $k \in \mathbb{N}$. Hence, $\lim\limits_{k\to\infty} d_k = u \geqslant 0$. We claim $u = 0$. Suppose $u \neq 0$, and taking the limit superior in inequality (2), we have

$$u \leqslant \limsup_{k\to\infty} \phi\left((d_{k-1})^{\alpha+\beta}(d_k)^{\gamma+1-\lambda}\right) \leqslant \phi(u),$$

which contradicts the fact that $\phi(t) < t$ for all $t > 0$. Hence, $u = 0$. Next, we aim to prove that $(x_{2k})_{k\geqslant 0}$ is a Cauchy sequence. Assume, for the indirect proof, that it is not a Cauchy sequence. Then, by Lemma 2, there exist $\varepsilon > 0$ and a subsequences $(m(k))$,$(n(k))$, such that $m(k) > n(k) > k$,

$$P(x_{2m(k)-2}, x_{2n(k)}) \leqslant \varepsilon \quad \text{and} \quad P(x_{2m(k)}, x_{2n(k)}) > \varepsilon.$$

Put $x = x_{2m(k)-1}$,$y = x_{2n(k)-1}$ in inequality (1); then we have

$$\varepsilon \leqslant P(x_{2m(k)}, x_{2n(k)}) \leqslant \phi\Big((P(x_{2m(k)-1}, x_{2n(k)-1}))^\alpha (d_{2m(k)-1})^\beta (d_{2n(k)-1})^\gamma$$
$$(\frac{1}{2}(P(x_{2m(k)}, x_{2n(k)-1}) + d_{2n(k)-1} - P(x_{2n(k)-1}, x_{2n(k)-1})))^{1-\lambda}\Big) \leqslant$$
$$\leqslant \phi\Big((sd_{2m(k)-1} + s^2 P(x_{2m(k)}, x_{2n(k)}) + s^2 d_{2n(k)-1})^\alpha (d_{2m(k)-1})^\beta (d_{2n(k)-1})^\gamma$$
$$(\frac{1}{2}(sP(x_{2m(k)}, x_{2n(k)}) + (s+1)d_{2n(k)-1} - P(x_{2n(k)-1}, x_{2n(k)-1})))^{1-\lambda}\Big).$$

By taking the limit superior and using the upper semi-continuity of ϕ, we obtain $\varepsilon = 0$, leading to a contradiction. Hence, $(x_{2k})_{k\geqslant 0}$ is a Cauchy sequence. Now, we will show that $(x_k)_{k\geqslant 0}$ is a Cauchy sequence. Let $m \geqslant n$ for all $m, n \in \mathbb{N}$. If $n = 2k+1$, $m = 2l+1$, $k, l \geqslant 0$, then,

$$\begin{aligned} P(x_{2k+1}, x_{2l+1}) &\leqslant s(P(x_{2k+1}, x_{2k}) + P(x_{2k}, x_{2l+1})) \leqslant \\ &\leqslant sP(x_{2k+1}, x_{2k}) + s^2P(x_{2k}, x_{2l}) + s^2P(x_{2l}, x_{2l+1}). \end{aligned}$$

Hence, $\lim\limits_{n,m\to\infty} P(x_m, x_n) = 0$. Moreover, if $n = 2k$, $m = 2l+1$, then

$$P(x_{2k}, x_{2l+1}) \leqslant s(P(x_{2k}, x_{2l}) + P(x_{2l}, x_{2l+1})),$$

which further implies that $\lim\limits_{n,m\to\infty} P(x_m, x_n) = 0$. Since (X, P, s) is complete, there exists $x \in X$, such that $x_n \to x$ as $n \to \infty$. Note that $P(x, x) = \lim\limits_{n,m\to\infty} P(x_n, x_m) = \lim\limits_{n\to\infty} P(x_n, x)$. Now, it remains to show that x is the unique fixed point. Note that

$$P(x, Tx) \leqslant s(P(x, x_n) + P(x_n, T(x))). \tag{3}$$

Also,

$$\begin{aligned} P(x_n, Tx) \leqslant \phi\Big(&(P(x_{n-1}, x))^{\alpha}(d_{n-1})^{\beta}(P(x, Tx))^{\gamma} \\ &\big(\frac{1}{2}(P(x_n, x) + P(x, T(x)) - P(x, x))\big)^{1-\lambda}\Big). \end{aligned}$$

Hence,

$$\begin{aligned} \limsup_{n\to\infty} P(x_n, T(x)) \leqslant \limsup_{n\to\infty} \phi\Big(&(P(x_{n-1}, x))^{\alpha}(d_{n-1})^{\beta}(P(x, Tx))^{\gamma} \\ &\big(\frac{1}{2}(P(x_n, x) + P(x, Tx) - P(x, x))\big)^{1-\lambda}\Big) \leqslant \phi(0) = 0. \end{aligned}$$

With reference to inequality (3), we have

$$P(x, Tx) \leqslant \limsup_{n\to\infty} \left(s(P(x, x_n) + P(x_n, Tx))\right) = 0,$$

which implies $P(x, Tx) = 0$, thus $x = Tx$. Let y be another fixed point of T. Then

$$\begin{aligned} P(x, y) \leqslant \phi\Big(&(P(x, y))^{\alpha}(P(x, Tx))^{\beta}(P(y, Ty))^{\gamma} \\ &\big(\frac{1}{2}(P(Tx, y) + P(y, Ty) - P(y, y))\big)^{1-\lambda}\Big), \end{aligned}$$

which yields $P(x,y) \leqslant 0 = 0$. Hence, $x = y$. □

Example 1. Let $X = \{1,2,3,4\}$ and $P\colon X \times X \to \mathbb{R}$ be defined as

$$P(x,y) = \begin{cases} |x-y|^2 + \max(x,y), \text{ when } x \neq y, \\ x, \text{ when } x = y \neq 1, \\ 0, \text{ else.} \end{cases}$$

Then, (X, P, s) is a complete partial b-metric space with coefficient $s = 4$. Define $T\colon X \to X$ given by

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 1 & 2 & 2 \end{pmatrix}.$$

Define $\psi\colon [0,\infty) \to [0,\infty)$ by

$$\psi(t) = \frac{t}{2^{\gamma}}, \quad \text{for all } t \geqslant 0.$$

The functions P, T and ψ defined above satisfy inequality (1) for all points $(x,y) \in X \times X$, provided that $\alpha = 0.1, \beta = 0.87$ and $\gamma = 0.02$. This is demonstrated in the following table:

(x, y)	P(Tx, Ty)	Value of R.H.S of inequality (1)
(2,2)	0	2.98
(2,4)	3	3.34
(3,2)	3	3.86
(4,3)	2	7.29
(3,4)	2	3.15
(4,4)	2	7.29
(2,3)	3	3.05
(3,3)	2	3.79

Thus, by Theorem 3, 1 is the only fixed point and $P(1,1) = 0$.

3. Interpolative Matkowski type contraction and fixed-point theorem. In this section, we introduce an interpolative Matkowski contraction in the setting of partial b-metric space inspired by the works of Aydi et al. [1] on these contractions in metric spaces. We begin with the following definition:

Definition 10. *Let (X, P, s) be a partial b-metric space with coefficient $s \geqslant 1$. Then the operator T from X to X is called an interpolative Matkowski type contraction in partial b-metric space, if there are $\alpha, \beta, \gamma \in (0, 1)$, such that $\lambda = \alpha + \beta + \gamma < 1$ and $\psi \in \mathcal{G}$ with*

$$P(Tx, Ty) \leqslant \psi\Big([P(x, y)]^{\alpha}[P(x, Tx)]^{\beta}[P(y, Ty)]^{\gamma}[\frac{1}{2}(P(Tx, y)+ \\ + P(y, Ty) - P(y, y))]^{1-\lambda}\Big) \text{ for all } x, y \in X\backslash Fix(T). \quad (4)$$

Remark. *The expression in the right-hand side of the inequality (4) is non-negative and not symmetric in x and y.*

Theorem 4. *Let (X, P, s) be a complete partial b-metric space with coefficient $s \geqslant 1$ and T be an interpolative Matkowski type contraction. Then T has a unique fixed point x (say) in X and $P(x, x) = 0$.*

Proof. Let $x_0 \in X$. Define $T^k x_0 = x_k$ for every $k \in \mathbb{N} \cup \{0\}$. If $x_k = x_{k+1}$, for some $k \in \mathbb{N}$, then x_k becomes a fixed point of T. Assume that $x_k \neq x_{k+1}$, for all $k \in \mathbb{N} \cup \{0\}$. Put $x = x_{k-1}$ and $y = x_k$ in (4) to get

$$P(x_k, x_{k+1}) = P(Tx_{k-1}, Tx_k) \leqslant \psi\Big((P(x_{k-1}, x_k))^{\alpha}(P(x_{k-1}, x_k))^{\beta} \\ (P(x_k, x_{k+1}))^{\gamma}(\frac{1}{2}(P(x_k, x_k) + P(x_k,x_{k+1}) - P(x_k, x_k)))^{1-\lambda}\Big),$$

where $\alpha, \beta, \gamma \in (0, 1)$ and $\lambda = \alpha + \beta + \gamma < 1$.
Let $d_k = P(x_k, x_{k+1})$. Then we have

$$d_k \leqslant \psi\Big((d_{k-1})^{\alpha+\beta}(d_k)^{\gamma}(\frac{1}{2}(d_k))^{1-\lambda}\Big) \leqslant \psi\Big((d_{k-1})^{\alpha+\beta}(d_k)^{\gamma+1-\lambda}\Big).$$

If $d_k > d_{k-1}$ for some $k \in \mathbb{N}$, then $d_k \leqslant \psi(d_k)$. Now, as $\psi \in \tau$, $d_k < d_k$, which is absurd. Hence, $d_k \leqslant d_{k-1}$ for all $k \in \mathbb{N}$ and $\lim_{k\to\infty} d_k = u \geqslant 0$. Next, we claim that $u = 0$. As $d_n \leqslant \psi(d_{n-1}) \leqslant \psi^n(d_0)$, we get $u = \lim_{n\to\infty} d_n \leqslant \lim_{n\to\infty} \psi^n(d_0) = 0$. Hence, $u = 0$. The remaining part of the proof follows same as in Theorem 3. □

Example 2. Define $\psi \colon [0, \infty) \to [0, \infty)$ by

$$\psi(x) = \begin{cases} \dfrac{x}{(2.1)^{\gamma}}, & \text{when } x < 1, \\ \dfrac{x}{2^{\gamma}}, & \text{else.} \end{cases}$$

For $\gamma = 0.02$, we find that ψ is non-decreasing. Also, $\lim_{k\to\infty} \psi^k(x) = 0$ for each $x \in [0, \infty)$. Thus, $\psi \in \mathcal{G}$. Note that for each $x \in X$, we get $\psi(x) = \dfrac{x}{2^\gamma}$. Therefore, we can easily demonstrate that it is an interpolative Matkowski contraction under the same conditions as in the previous example, by replacing ϕ with ψ.

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