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R. F. Shamoyan, S. M. Kurilenko

ON TRACES OF ANALYTIC HERZ AND BLOCH TYPE SPACES IN BOUNDED STRONGLY PSEUDOCONVEX DOMAINS IN $\mathbb{C}^n$

Abstract. In our paper we provide some direct extentions of our recent sharp results on traces in the analytic function spaces, which we proved earlier in case of the unit ball in $\mathbb{C}^n$, to the case of the bounded strongly pseudoconvex domains with a smooth boundary. To be more precise we consider the analytic Bloch space in the strongly pseudoconvex domains with a smooth boundary, mixed norm spaces and so-called the new Herz type spaces of analytic functions in the domains of the same type. The Bloch spaces, for various complicated domains, were studied by many authors, but the various Herz type spaces are introduced in this paper, as far as we know, for the first time. The role of so-called r-lattices and their new properties are essential for our proofs. These techniques based on the lattices in the strongly pseudoconvex domains were invented and heavily used in the recent papers of Abate and coauthors. The arguments in the proofs in the case of the unit ball and the strongly pseudoconvex domains have some similarity.

Key words: *pseudoconvex domains, analytic functions, mixed-norm spaces, Herz-type spaces, Bloch spaces*

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Introduction. Let $D = \{z : \rho(z) < 0\}$ be a bounded strongly pseudoconvex domain of $\mathbb{C}^n$ with C^∞ boundary. We assume that the strongly plurisubharmonic function ρ is of class C^∞ in a neighborhood of $\overline{D}$, that, $-1 \le \rho(z) < 0, z \in D$ and $|\partial\rho| \ge c_0 > 0$ for $|\rho| \le r_0$.

Denote by $\mathcal{O}(D)$ or $(H(D))$ the space of all analytic functions on D. Denote (see [1]) $A^{p,q}_{\delta,k}(D) = \{f \in H(D) : \|f\|_{p,q,\delta,k} < \infty\}$, where

$$\|f\|_{p,q,\delta,k} = \left(\sum_{|\alpha| \le k} \int_0^{r_0} \left(\int_{\partial D_r} |\mathcal{D}^\alpha f|^p d\sigma_r \right)^{q/p} r^{\delta \frac{q}{p} - 1} dr \right)^{1/q},$$

here $D_r = \{z \in \mathbb{C}^n : \rho(z) < (-r)\}$, ∂D_r is a boundary, $d\sigma_r$ is a normalized surface measure on ∂D_r and by dr we denote a normalized volume element on $(0, r)$, $0 < p < \infty$, $0 < q \le \infty$, $\delta > 0$, $k = 0, 1, 2...$ and

$$\|f\|_{p,\infty,\delta,k} = \sup \left\{ \left(\sum_{|\alpha| \le k|} (r^\delta) \int_{\partial D_r} |\mathcal{D}^\alpha f|^p d\sigma_r \right)^{1/p} : 0 < r < r_0 \right\}$$

(for $p, q < 1$ it is quazinorm) (see [1]), where $\mathcal{D}^\alpha$ is a derivative of f (see [1]). It can be easily shown these spaces are the Banach spaces for $\min(p, q) \ge 1$. Further, for $p = q$ we have

$$\|f\|_{p,\delta,k} = \left(\sum_{|\alpha| \le k} \int_D |\mathcal{D}^\alpha f(\zeta)|^p (-\rho(\zeta))^{\delta - 1} dv(\zeta) \right)^{1/p}; \ \delta > 0, \ k \ge 0,$$

where $dv(\zeta)$ (or sometimes $dV(\zeta)$) is a normalized Lebegues measure on D (for $k = 0$ we will use another, more convenient, notation for A^p_α spaces). Some interesting properties of these classes can be seen in lemmas presented below.

As we see for $p = q$; $k = 0$, we get the usual Bergman spaces $A^p_\delta(D)$. One of the goals of this paper is to consider a new trace map in the trace problem for the case of the bounded strongly pseudoconvex domains with a smooth boundary; it is a map $T_R f(z) = f(z, ..., z)$, $z \in D$ for $f \in X \subset \mathcal{O}(D^m)$ for a certain quazinormed analytic space X on D^m.

These type of maps (diagonal map) were previously considered by various authors in particular cases when $\Omega = D$ (unit disk), or when $\Omega = B$ (unit ball), (see for example [2], [3], [4] and various references there). Applications of this map to various problems in the complex function theory are also known (see, for example [2], [3], [5] and references there). Our paper contains various remarks. We alert the reader that some points related to a calculation of indexes are missed since they are similar to the unit ball case.

The structure of this paper is the following. The first section contains some assertions and preliminaries useful in the following. We collect the new sharp assertions on a Trace type operator in section 2. We actively use some machinery which was recently found and developed in [1], [6], [7].

Some assertions of this paper were taken from our previous papers [8], [4], [2], [9], where some results were proved in a less general situation namely in a case of the unit ball in $\mathbb{C}^n$ (the most simple model of domain D is considered in this paper).

One of the goals of this paper is to generalize them to the arbitrary bounded pseudoconvex domains with a smooth boundary. We mention separately [10], [11] where complete descriptions of traces of the other Bergman type spaces in bounded pseudoconvex domains with a smooth boundary were also obtained.

This paper, in particular, is devoted to the problem of description of traces of the so-called Herz-type spaces. Finally, we denote various positive constants in this paper by C or by C with the lower indexes. It worth to note that the analytic function spaces in bounded strongly pseudoconvex domains were intensively studied in recent decades (see for example [1] and [6, 7] and references there). Finally, we mention a big series of papers of the first author on traces of the analytic functions in the unit ball (see [2, 4, 9, 10, 11, 8]. New trace theorems and related estimates on the tubular domains over symmetric cones are presented in [12, 13, 14].

1. Preliminary lemmas. In this section we collect some known lemmas and theorems on pseudoconvex domains, some of them we will need later. Since the estimates and assertions were previously known, the appropriate citation will be also found below. Let Ω be a bounded domain in C^n. Let N be a real vector field in a neighborhood of $\partial\Omega$ which agrees with the outward unit normal vector field on $\partial\Omega$. It is known that for $z \in \partial\Omega$ and $t > 0$ sufficiently small, say $0 < t < \delta_0$, the integral curve of N through z has a unique intersection point with the hypersurface $\{\delta = t\}$.

For $f \in \mathcal{O}(\Omega)$, denote

$$(\mathcal{D}^\alpha f)(z) = \frac{\partial^{|\alpha|} f}{\partial z_1^{\alpha_1} \dots \partial z_n^{\alpha_n}}(z).$$

Let $\mathcal{N}$ be the complex normal vector field of type $(0,1)$ defined by

$$\mathcal{N} = \sum_{j=1}^{n} \frac{\partial \varrho}{\partial \bar{\zeta}_j} \frac{\partial}{\partial \zeta_j} \qquad (\textit{see } [1]).$$

Below we use the notation $A \lesssim B$ for the two expressions A and B which means that there is a constant C, independent of the quantities under consideration, such that $A \leq C \cdot B$. When $A \lesssim B$ and $B \lesssim A$, we use the notation $A \sim B$.

We provide basic facts on the Henkin–Ramirez function $\tilde{\Phi}$ based on [15], then we will use it to define the Bergman kernel.

Let Ω be a C^∞-bounded strongly pseudoconvex domain with a function ρ defined on it. We need some well known defintions and results for our proofs (see [15] and [16] and references there for more details).

Let $g(z,\zeta)$ be the associated Levi polynomial

$$g(z,\zeta) = 2\sum_{j=1}^{n} \frac{\partial \rho}{\partial \zeta_j}(\zeta)(\zeta_j - z_j) - \sum_{j,k=1}^{n} \frac{\partial^2 \rho}{\partial \zeta_j \partial \zeta_k}(\zeta)(\zeta_j - z_j)(\zeta_k - z_k).$$

It follows from Taylor's formula and the strict plurisubharmonicity of ρ that there are positive constants C_1 and r and a neighborhood Ω' of $\bar{\Omega}$ such that

$$Re\ g(z,\zeta) \geq \rho(\zeta) - \rho(z) + C_1|z-\zeta|^2$$

for $z,\zeta \in \Omega'$ and $|z-\zeta| \leq r$. Setting $\tilde{g}(z,\zeta) = g(z,\zeta) - 2\rho(\zeta)$, we obtain

$$Re\ \tilde{g}(z,\zeta) = Re\ g(z,\zeta) - 2\rho(\zeta) \geq -\rho(\zeta) - \rho(z) + C_1|z-\zeta|^2 \qquad (1)$$

for $z,\zeta \in \Omega'$ and $|z-\zeta| \leq r$ and $\tilde{g}(z,\zeta) = g(z,\zeta)$ for $\zeta \in \partial\Omega$. Also we have

$$\mathcal{N}\tilde{g}(z,\zeta) = \mathcal{O}(|z-\zeta|^2).$$

Lemma 1. [15, 22] *Let $\tilde{g}$, Ω', r and C_1 be as above. There is a neighborhood $\tilde{\Omega}$ of $\bar{\Omega}$ with $\tilde{\Omega} \supset \Omega'$, a C^∞-function $\tilde{\Phi}$ on $\tilde{\Omega} \times \tilde{\Omega}$, and a positive constant C_2 such that*

- *(i) for any $\zeta \in \tilde{\Omega}$ the function $\tilde{\Phi}(\cdot,\zeta)$ is holomorphic on $\tilde{\Omega}$;*
- *(ii) $\tilde{\Phi}(\zeta,\zeta) = -2\rho(\zeta)$ for $\zeta \in \tilde{\Omega}$, and $|\tilde{\Phi}(z,\zeta| \geq C_2$ for $z,\zeta \in \tilde{\Omega}$ with $|z-\zeta| \geq \frac{r}{2}$;*

(iii) there is a non-vanishing C^∞ function $Q(z,\zeta)$ on

$$\Delta_{\frac{r}{2}} = \left\{(z,\zeta) \in \tilde{\Omega} \times \tilde{\Omega} : |z-\zeta| \le \frac{r}{2}\right\}$$

such that $\tilde{\Phi}(z,\zeta) = \tilde{g}(z,\zeta)Q(z,\zeta)$ on $\Delta_{\frac{r}{2}}$.

Lemma 2. [15] *For each $s > -1$, there is a smooth form $\eta_s \in C^\infty(\tilde{\Omega} \times \tilde{\Omega})$ such that*

(i) $\eta_s(z,\zeta)$ is holomorphic in z on $\tilde{\Omega}$ for any fixed $\zeta \in \tilde{\Omega}$, and

(ii) for $f \in A^1_s(\Omega)$ and $z \in \Omega$ we have

$$f(z) = \int_\Omega f(\zeta)\frac{\eta_s(z,\zeta)}{\tilde{\Phi}(z,\zeta)}(-\rho(\zeta))^s dV(\zeta)$$

We mention a more general version of this lemma as well.

Lemma 3. *Suppose $s > -1$ and let m be a non-negative integer. Then, for $f \in A^1_s$ we have the representation*

$$f(z) = \sum_{j=0}^{m} \int_\Omega \mathcal{N}^j f(\zeta)\frac{\eta_{s+j}(z,\zeta)}{\tilde{\Phi}(z,\zeta)^{n+s+1}}(-\rho(\zeta))^{s+m} dV(\zeta).$$

Now, we define the Bergman type kernels now via the Henkin–Ramirez function.

Definition 1. [15] *We say that K is a Bergman kernel of type t, $t > 0$, and we will write K_t, if $|K(z,\zeta)| \le c|\tilde{\Phi}(z,\zeta)|^{-t}$ where $\tilde{\Phi}$ is a special function (the Henkin–Ramirez function). So if K is a kernel of type t, then K^s is a kernel of type st, and we denote it by K_t, where $t > 0$.*

The reproducing kernel for the weighted Bergman space A^2_t is $K_{n+t+1}(z,w)$, and for the unweighted Bergman space A^2 is K_{n+1}.

Throughout this section and the next, D will denote a bounded strongly pseudoconvex domain in $\mathbb{C}^n$ with a smooth boundary. We fix the plurisubharmonic characterizing function ρ for D, and let $g(z,\zeta)$ be the associated Levi polynomial (see [6], [7]). We shall use, in particular, the following notations in pseudoconvex domains:

- $\delta\colon D \to R^+$ will denote the Euclidean distance from the boundary, that is $\delta(z) = d(z,\partial D)$;

- given $0 < p \leq +\infty$, the *Bergman space* $A^p(D)$ is the Banach space $L^p(D) \cap \mathcal{O}(D)$, endowed with the L^p-norm;
- more generally, given $\beta \in R$, we introduce the *weighted Bergman space*
$$A^p(D, \beta) = L^p(\delta^\beta v) \cap \mathcal{O}(D)$$
endowed with the quazinorm
$$\|f\|_{p,\beta} = \left[\int_D |f(\zeta)|^p \delta(\zeta)^\beta \, dv(\zeta)\right]^{1/p}$$
if $0 < p < \infty$, and with the norm
$$\|f\|_{\infty,\beta} = \|f\delta^\beta\|_\infty$$
if $p = \infty$;
- $v_t(\zeta) = \delta^t(\zeta) dv(\zeta)$, $t > -1$;
- for each $z_0 \in D$ we shall denote by $k_{z_0} \colon D \to C$ the *normalized Bergman kernel* defined by
$$k_{z_0}(z) = \frac{K(z, z_0)}{\sqrt{K(z_0, z_0)}} = \frac{K(z, z_0)}{\|K(\cdot, z_0)\|_2} \ ;$$
- given $r \in (0, 1)$ and $z_0 \in D$, we shall denote by $B_D(z_0, r)$ the Kobayashi ball of center z_0 and radius $\frac{1}{2} \log \frac{1+r}{1-r}$.

Let us now recall a number of vital results proved in D. The first two give vital information about the shape of Kobayashi balls:

Lemma 4. (see [6], Lemma 2.1) *Let $D \subset\subset C^n$ be a bounded strongly pseudoconvex domain, and $r \in (0, 1)$. Then*
$$\nu\big(B_D(\cdot, r)\big) \approx \delta^{n+1} \ ,$$
where the constant depends on r.

Lemma 5. (see [6], Lemma 2.2) *Let $D \subset\subset C^n$ be a bounded strongly pseudoconvex domain. Then there is $C > 0$ such that*
$$\frac{C}{1-r}\delta(z_0) \geq \delta(z) \geq \frac{1-r}{C}\delta(z_0)$$
for all $r \in (0, 1)$, $z_0 \in D$ and $z \in B_D(z_0, r)$.

We shall also need the existence of suitable coverings by Kobayashi balls:

Definition 2. *Let $D \subset \mathbb{C}^n$ be a bounded domain, and $r > 0$. An* r-lattice *in D is a sequence $\{a_k\} \subset D$ such that $D = \bigcup_k B_D(a_k, r)$ and there exists $m > 0$ such that any point in D belongs to at most m balls of the form $B_D(a_k, R)$, where $R = \frac{1}{2}(1 + r)$.*

The existence of r-lattices in bounded strongly pseudoconvex domains is ensured by the following

Lemma 6. (see [6], Lemma 2.5) *Let $D \subset \mathbb{C}^n$ be a bounded strongly pseudoconvex domain. Then for every $r \in (0, 1)$ there exists an r-lattice in D, i. e. there exist $m \in \mathbf{N}$ and a sequence $\{a_k\} \subset D$ of points such that $D = \bigcup_{k=0}^{\infty} B_D(a_k, r)$ and no point of D belongs to more than m of the balls $B_D(a_k, R)$, where $R = \frac{1}{2}(1 + r)$.*

We shall use a vital submean estimate for the non-negative plurisubharmonic functions on the Kobayashi balls:

Lemma 7. (see [6], Corollary 2.8) *Let $D \subset\subset \mathbb{C}^n$ be a bounded strongly pseudoconvex domain. Given $r \in (0, 1)$, set $R = \frac{1}{2}(1 + r) \in (0, 1)$. Then there exists $C_r > 0$ depending on r such that*

$$\forall z_0 \in D \; \forall z \in B_D(z_0, r) \qquad \chi(z) \leq \frac{C_r}{\nu\left(B_D(z_0, r)\right)} \int_{B_D(z_0, R)} \chi \, dv$$

for every non-negative plurisubharmonic function $\chi \colon D \to \mathbf{R}^+$.

A version of lemma 3 is valid also for D^α derivatives. These type of assertions play an important role in various theorems in complex function theory (see [1], for example). We formulate such version below.

Lemma 8. [1] *Let s be a nonnegative integer. There exist kernels $K^1_\alpha(\zeta, z)$, $|\alpha| \leq k$, each holomorphic with respect to z such that*

$$f(z) = \sum_{|\alpha| \leq k} \int_D \mathcal{D}^\alpha f(\zeta) \tilde{K}^1_\alpha(\zeta, z) (\rho(\zeta)^{s+k} dv(\zeta)$$

for every function f of $A^1_{s,k}$ class and $|\mathcal{D}^\beta \tilde{K}^1_\alpha(\zeta, z)| \leq c_\beta |a(\zeta, z)|^{-(n+1+s+[\beta])}$ for each multi-index β, $z \in D$ and for some special function a.

Lemma 9. [1] *For $s > -1$, and $t \in R$*

$$\int_D (-\rho(\zeta))^s |a(\zeta, z)|^t dv(\zeta) = \begin{cases} 1 \text{ if } n+1+s+t > 0; \\ \lg|\rho(z)| \text{ if } n+1+s+t = 0; \\ |\rho(z)|^{n+1+s+t} \text{ if } n+1+s+t < 0. \end{cases}$$

The same estimate is valid if we replace $|a|^t$ function by K_t Bergman type kernel, $t > 0$ (see, for example, [17] and [6], [18]).

Lemma 10. [18] *For $\Phi \in H(D)$, $p \leq 1$, $\beta > -1$, $\alpha \in \mathbb{N}$, $m \in \mathbb{N}$. Then*

$$\left(\int_D |\Phi(z)| \prod_{k=1}^m |K_\alpha(z, z_k)| \delta^\beta(z) dv(z) \right)^p \leq$$

$$\leq c \int_D |\Phi(z)|^p \delta(z)^{\beta p + p(n+1) - (n+1)} \prod_{k=1}^m |K_\alpha(z, z_k)|^p dv(z)$$

$z_k \in D$, $k = 1, \ldots, m$.

This lemma is well known in the unit disk and in the unit ball (see, for example, [3], [4]). This lemma is also known for m=1 (see [11] in case of general bounded pseudoconvex domains with a smooth boundary).

If $a_k \in D$ then $|K_t(a_k, w)| \asymp |K_t(a_k, a_k)|$, $w \in B_D(a_k, r)$, $t = s(n+1)$, $s \in \mathbb{N}$ (see [19]). In this paper we assume for all $t > 0$

$$|K_t(z, w)| \asymp |K_t(z, a_k)|, \quad w \in B_D(a_k, r), \quad z \in D.$$

This is valid in the unit ball (see [4], [8]). In [10], [11] this condition is not needed.

2. On traces in analytic spaces in bounded strongly pseudoconvex domains and some related estimates for the Bergman-type projections in $\mathbb{C}^n$. The main section of the article is devoted to formulations and proofs of all main results of this paper related to sharp trace theorems in the new analytic Herz type spaces and Bloch type spaces in bounded strongly pseudoconvex domains with a smooth boundary in $\mathbb{C}^n$. Trace theorems have many applications in various problems in complex function theory (see, for example [3], [5] and various references there).

Let $H(D^m)$ be the space of all analytic functions in D^m, $m \in \mathbb{N}$. We say $Trace\ X = Y$, if X is a quazinormed subspace of $H(D^m)$ and Y is a certain fixed quazinormed subspace of $H(D)$, if for each F, $F \in X$,

$F(z,\dots,z)=f(z)$, $f\in Y$ and the reverse is also true: for each g, $g\in Y$, there is a function F, $F\in X$, such that $F(z,\dots,z)=g(z)$, for all $z\in D$.

Lemmas from previous section allow to prove several new sharp assertions on traces (we use, in particular, the machinery taken from recent papers [6], [7], [17] related with r-lattices and repeat arguments we provided earlier in the unit ball case (see [2], [10], [11, 9, 8]).

Let $dv(z_1,\dots,z_m)=\prod_{j=1}^m dv(z_j)=dv(\vec{z})$, $z_j\in D$, $j=1,\dots,m$ be the normalized Lebeques measure on a product domain. We denote, as usual, by $dv_\gamma=\delta^\gamma(z)dv(z)$ the weighted Lebegues measure on D and similarly on products of such domains using products of δ functions in a standard way. We define new analytic Herz-type spaces on products of pseudoconvex domains with a smooth boundary in $\mathbb{C}^n$ via Kobayashi balls as follows (using a standard sign of a vector on lower indexes below, we get a little bit more general versions of these spaces, and this is a standard extension in literature).

Let it further be

$$K^{p,q}_{\alpha,\beta}(D^m)=\{f\in H(D^m):$$

$$\int_D\cdots\int_D\left(\int_{B_D(\tilde{z}_1,R)}\cdots\int_{B_D(\tilde{z}_m,R)}|f(z_1,\dots,z_m)|^p\times\right.$$

$$\times\delta^\alpha(z_1)\dots\delta^\alpha(z_m)\times dv(z_1,\dots,z_m))^{q/p}\prod_{k=1}^m\delta^\beta(\tilde{z}_k)dv(\tilde{z}_1,\dots,\tilde{z}_m)<\infty,$$

$p,q\in(0,\infty)$, $\alpha>-1$, $\beta>-1\}$.

In less general cases of the unit disk, polydisk, the unit ball the spaces of such type were studied in [8], [9], [10], [11]. These are the Banach spaces for $min(p,\ q)\ge 1$ and complete metric spaces for the other values of p and q. The natural question is, can each $f\in A^p_\alpha(D)$ be extended to an analytic F function, so that $F(z,\dots,z)=f(z)$, $z\in D$ and $F\in K^{p,q}_{t,\beta}(D^m)$ with some restrictions on parameters involved. We fix r-lattice $\{a_k\}\in D$. For the same values of parameters, we define another analytic Herz type space in pseudoconvex domains with a smooth boundary as follows:

$$K^{p,q}_{\beta}(D^m)=\{f\in H(D^m):$$

$$\sum_{k_1}\cdots\sum_{k_m}\left(\int_{B(a_{k_1},r)}\cdots\int_{B(a_{k_m},r)}|f(z_1,...z_m)|^p(\delta_(z_1)^\beta\dots\right.$$

$$\dots (\delta(z_m))^{\beta} dv(z_1, \dots, z_m)\Big)^{q/p} < \infty,$$
$$0 < p, q < \infty, \beta > -1\}.$$

If $p = q$, we get the classic Bergman spaces on the products of the pseudoconvex domains with a smooth boundary. This simple fact is based on the properties of r-lattices. These new classes of analytic functions are the Banach spaces for all values of p, q so that $\min(p, q) > 1$ and complete metric spaces for other values of parameters. It is naturally to calculate traces of these spaces. Looking at quazinorms above, we can replace the outer product integration or product sum by one integral or one sum defining similar spaces with the following quazinorms:

$$\sum_k \left(\int_{B(a_k,r)} \cdots \int_{B(a_k,r)} |f(z_1, ...z_m)|^p (\delta_(z_1)^{\beta} \dots (\delta(z_m))^{\beta} \times\right.$$
$$\times dv(z_1, \dots, z_m))^{q/p} < \infty,$$
$$0 < p, q < \infty, \ \beta > -1,$$
$$\int_D \left(\int_{B_D(\tilde{z},R)} \cdots \int_{B_D(\tilde{z},R)} |f(z_1, \dots, z_m)|^p \delta^{\alpha}(z_1) \dots \delta^{\alpha}(z_m) \times\right.$$
$$\times dv(z_1, \dots, z_m))^{q/p} \delta^{\beta}(\tilde{z}) dv(\tilde{z}) < \infty, \qquad p, q \in (0, \infty), \alpha > -1, \beta > -1.$$

Traces of these new analytic spaces can be also calculated similarly using modifications of methods applied in this paper.

The following theorem is one of the main results of this paper.

Theorem 1. *Let* $0 < p < \infty$, $t_j > -1$, $\beta_j > -1$, $j = 1, \dots, m$, $\alpha > -1$,

$$\alpha_2 = \sum_{j=1}^{m} (\beta_j + 2(n+1) + t_j) - (n+1).$$

Then, if $f \in K^{p,p}_{t,\beta}(D^m)$, *then* $f(z, \dots, z) \in A^p_{\alpha_2}(D)$ *and*

$$\int\limits_D |f(z, \dots, z)|^p \delta^{\alpha_2}(z) dv(z) < \infty;$$

and the reverse assertion is also true. So $Trace K^{p,p}_{t,\beta}(D^m) = A^p_{\alpha_2}$. *For the same values of parameters we have,*

$$Trace K^{p,p}_{\beta}(D^m) = A^p_{\alpha_3},$$

where

$$\alpha_3 = (\sum_1^m \beta_j) + (n+1)(m-1).$$

Moreover, we have the following inclusions.

$$Trace K^{q,p}_{\beta}(D^m) \subset A^p_{\alpha_1}(D)$$

and

$$Trace K^{q,p}_{\beta_t}(D^m) \subset A^p_{\alpha}(D),$$

where

$$\alpha_1 = \sum (\beta_j + (n+1))p/q - n - 1,$$

$$\alpha = p/q(\sum (\beta_j + n + 1)) + \sum (t_j + n + 1) - (n+1)$$

for all positive values of p *and* q*.*

Remark 1. *For* $m = 1$ *some of these assertions are obvious. For the unit ball case in* $\mathbb{C}^n$*, these assertions were proved in [2], [4], [11, 9, 8]. The complete analogues of some of these assertions are also valid for the tubular domains over symmetric cones (see [12], [13]).*

The proof of the Theorem 1 will be given in full below. It is parallel to the unit ball case. For the proof of the Theorem 2, we refer to [2] and, as we just mentioned, approaches we use are rather similar to the unit ball case.

Remark 2. *In our previous notes [10] and [11], we evaluated traces of* $A^p_\alpha(D \times \cdots \times D)$ *and also studied various properties of the expanded Bergman projection using from the recent papers of Abate and co-authors [6, 7] and the properties of new* r*-lattices they have invented recently.*

Remark 3. *As corollaries of our theorems using various embeddings relating* $A^{p,q}_{\delta,k}$ *spaces with each other for various indexes some not sharp trace results can be also provided (see [1], [15] for such type embeddings). To calculate completely traces of more general mixed norm* $A^{p,q}_{\delta,k}$ *spaces is a separate and rather interesting problem. This problem probably can be solved by modification of approaches we invented in this paper.*

The proof of the Theorem 1. Let us fix a concrete r-lattice for D. Using properties of r-lattice, mentioned in the first section, we have (by lemmas 4–7 and definition 2)

$$\int_D |f(z,\dots,z)|^p \delta^\alpha(z)dv(z) \le c \sum_{k\ge 0} \max_{z\in B_D(a_k,r)} |f(z,\dots,z)|^p \times$$

$$\times \int_{B_D(a_k,r)} \delta^\alpha(w)dv(w).$$

Let

$$c_{k,\alpha} = \int_{B_D(a_k,r)} \delta(w)^\alpha dv(w).$$

Then we have

$$\sum_{k\ge 0} \max_{z\in B_D(a_k,r)} |f(z,\dots,z)|^p c_{k,\alpha} \lesssim \sum_{\substack{k_1\ge 0\\ \dots\\ k_m\ge 0}} \max_{z_j\in B_D(a_k,r)} |f(z_1,\dots,z_m)|^p \times$$

$$\times (c_{k_1,\alpha})^{\frac{1}{m}} \cdot \dots \cdot (c_{k_m,\alpha})^{\frac{1}{m}}. \qquad (A)$$

Let us assume that

$$\alpha = \frac{p}{q}(\sum_{j=1}^m \tilde{\beta}_j) + \sum_{j=1}^m t_j - \frac{p}{q}(n+1), t_j > -1,\ j = 1,\dots,m,$$

$$\tilde{\beta}_j = \beta_j + 2(n+1), j = 1,\dots,m.$$

We have followed closely the unit ball proof and used the properties of the Kobayashi balls. In more details, using (A) in the last estimate and m-times lemma 7, we have

$$J \asymp \int_D |f(w,\dots,w)|^p (\delta(w))^{(m-1)(n+1)+\sum_{j=1}^m s_j} dv(w) \le \qquad (2)$$

$$\le c_1 \sum_{k\ge 0} \int_{B_D(a_k,r)} |f(w,\dots,w)|^p)(\delta(w))^{(m-1)(n+1)+\sum_{j=1}^m s_j} dv(w) \le$$

$$\le c_2 \sum_{k\ge 0} (\sup_{w\in B_D(a_k,r)} |f(w,\dots,w)|^p) \times$$

$$\times \int_{B_D(a_k,r)} (\delta(w))^{(m-1)(n+1)+\sum_{j=1}^m s_j} dv(w) =$$

$$= c_3 \sum_{k\ge 0} (\sup_{w\in B_D(a_k,r)} |f(w,\dots,w)|^p) \delta(a_k)^t \delta(a_k)^{n+1} C(r) \le$$

$$\leq C_4 \|f\|^p_{A^p_{s_1,\dots s_m}}(D^m)$$

where $t = (m-1)(n+1) + \sum_{j=1}^m s_j$, $s_j > -1$ and $C(r)$ is constant (depends on r). This gives us a proof of the inclusion $Trace A^p_s(D^m) \subset A^p_v(D)$ for some parameters $s = (s_1, ...s_m)$ and v, where v depends on $s_1, ...s_m, m, n$. All we need is to modify this proof for our cases. From (2), after some small modifications, we obtain using lemmas 5–8

$$\|f\|^p_{A^p_{\alpha_1}} \leq C_1 \sum_{k_1 \geq 0} \cdots \sum_{k_m \geq 0} \left(\sup_{\substack{w_1 \in B_D(a_{k_1}, r) \\ \dots \\ w_m \in B_D(a_{k_m}, r)}} |f(w_1, \dots w_m)|^q \right)^{\frac{p}{/}q} \times$$

$$\times (\delta(a_{k_1}))^{\tau_1/m} \dots (\delta(a_{k_m}))^{\tau_m/m}$$

for some $\tau_1, \dots, \tau_m$.

Hence, using lemma 6 and lemma 7 for each variable, we have

$$\|f\|^p_{A^p_{\alpha_1}} < c \|f\|^p_{K^{q,p}_\beta}.$$

We have from (2) and from

$$\delta(a_k)^{\frac{p}{q}(n+1)+(n+1)} \sup_{z \in B(a_k, r)} |f(z)|^p \leq \int_{B(a_k, r)} |f(w)|^p dv(w) \delta(a_k)^{(n+1)\frac{p}{q}} \leq$$

$$\leq \int_{B(a_k, r)} \left(\int_{B(w, r)} |f(\tau)|^q dv(\tau) \right)^{\frac{p}{q}} dv(w), \ a_k \in D, \ k = 1, 2, \dots,$$

what we need.

Indeed,

$$\|f\|^p_{A^p_\alpha} \lesssim \int_D \cdots \int_D (\delta^{t_1}(\tilde{z}_1) \dots \delta^{t_m}(\tilde{z}_m)) \times$$

$$\times \left(\int_{B_D(\tilde{z}_1, r)} \cdots \int_{B_D(\tilde{z}_m, r)} |f(w_1, \dots, w_m)|^q \times \right.$$

$$\left. \times \delta^{\beta_1}(w_1) \dots \delta^{\beta_m}(w_m) d\vec{v}(w_1) \dots dv(w_m) \right)^{p/q} dv(\tilde{z}_1) \dots dv(\tilde{z}_m)$$

for all $0 < p, q < \infty$ and all $\beta_j > -1$ and all $t_j > -1$, $j = 1, ..., m$.

We used above the fact that (see [1]) $\delta^t(z) \asymp \delta^t(a_k)$, $t \in R$, $z \in$ $\in B_D(a_k, r)$ and again the properties of r-lattice (see lemmas 5–8). Let us now show the reverse estimate using lemmas obtained above. We again follow the arguments of the unit ball case. We concentrate only on $K^{p,p}_{\beta,t}$ spaces, the case of $K^{p,p}_{\beta}$ spaces is similar. If $f \in A^p_\alpha$, $0 < p \leq \infty$, $\alpha > -1$, then $f \in A^1_t(D)$ for large enough t and, hence, we have

$$f(z) = \int_D f(\zeta) K_{t+n+1}(z, \zeta) \delta(\zeta)^t dv(\zeta),$$

for all $z \in D$, where t is a large enough positive number (see [15]).

Then, let

$$F(z_1, \dots, z_m) = \int_D f(\zeta) \left(\prod_{j=1}^m K_{\frac{t+n+1}{m}}(z_j, \zeta) \right) \delta(\zeta)^t dv(\zeta),$$

$$z_j \in D, \ j = 1, \dots, m.$$

Then, using a remark above for a large enough t

$$F(z, \dots, z) = \int_D f(\zeta) \left(K_{\frac{t+n+1}{m}}(z, \zeta) \right)^m \delta(\zeta)^t dv(\zeta) =$$

$$= \int_D f(\zeta) \left(\tilde{K}_{t+n+1}(z, \zeta) \right) \delta(\zeta)^t dv(\zeta) = f(z),$$

$m \in \mathbb{N}$, $f \in A^p_\alpha$, $0 < p \leq \infty$, $\alpha > -1$. This fact is a vital ingredient in our proof.

We have now from lemma 10 for $q = p <= 1$

$$F(z_1, \dots, z_m) = c \int\limits_D f(w) \delta^s(w) \prod_{j=1}^m K_{\frac{t}{m}}(w, z_j) dv(w); \ t = s + 1 + n;$$

$$z_j \in D; \ j = 1 \dots, m.$$

$$F(z, \dots, z) = f(z); \ z \in D,$$

and s is large enough,

and

$$\int_D \cdots \int_D \prod_{k=1}^m \delta^{t_k}(\tilde{z}_k) \int_{B_D(\tilde{z}_1, r)} \cdots \int_{B_D(\tilde{z}_m, r)} |F(z_1, \dots, z_m)|^p \times$$

$$\times \prod_{k=1}^{m} \delta^{\beta_k}(z_k) dv(\vec{z}) dv(\tilde{z}) \leq$$

$$\leq c \int_D \cdots \int_D \int_{B_D(\tilde{z}_1, r)} \cdots \int_{B_D(\tilde{z}_m, r)} |f(w)| \delta(w)^{p(n+1+s)-(n+1)} \times$$

$$\times \prod_{j=1}^{m} |K_{\frac{t}{m}}(w, z_j)|^p \left(\prod_{k=1}^{m} \delta(\tilde{z}_k) \right)^{t_k} \prod_{k=1}^{m} \delta^{\beta_k}(z_k) dv(\vec{z}) d\tilde{v}(w).$$

From here using Fubbini's theorem, we use the Forrely-Rudin type estimate (see a remark after the lemma 9) $\|F\|^p_{K^{p,p}_{t,\beta}} \leq c\|f\|^p_{A^p_{\alpha_2}}$.

Using lemmas 4, 5 and condition on a kernel, we have

$$\int_D \cdots \int_D \left(\int_{B_D(\tilde{z}_1, r)} \cdots \int_{B_D(\tilde{z}_m, r)} \prod_{j=1}^{m} |K_{\frac{t}{m}}(w, z_j)|^p \times \right.$$

$$\left. \times \prod_{k=1}^{m} \delta(z_k)^{\beta_k} dv(z_1) \dots dv(z_m) \right) \times$$

$$\times \prod_{j=1}^{m} \delta(\tilde{z}_j)^{t_j} dv(\tilde{z_1}) \dots dv(\tilde{z}_m) \leq c\delta(w)^{\mathrm{v}}, \ w \in D,$$

$\mathrm{v} = \alpha_2 + (n+1) - p(n+1+s)$, $\alpha_2 = \sum_{j=1}^{m}(\beta_j + t_j) + (n+1)(2m-1)$, $t = s + n + 1$, $s > s_0$, s_0 is a large enough.

For $p > 1$ case, we refer to the unit ball case in our previous work mentioned above. The proof follows from the similar arguments, the Holder's inequality applied twice, lemmas 4–9, the estimates of the Bergman kernel of Forelly–Rudin type.

We have

$$|F(z_1, \dots, z_m)|^p \leq c \left(\int_D |f(w)|^p \prod_{j=1}^{m} |K_{\frac{t}{m}}(w_j, z_j)|^{\gamma_1 p} \delta(w)^s dv(w) \right) \times$$

$$\times \left(\int_D \delta(w)^s \prod_{j=1}^{m} |K_{\frac{t}{m}}(w_j, z_j)|^{\gamma_2 q} dv(w) \right)^{\frac{p}{q}} = G_1 G_2,$$

$\frac{1}{p} + \frac{1}{q} = 1$, $\gamma_1 + \gamma_2 = 1$.

Then, $G_2 \leq c\prod_{j=1}^{m} \delta(z_j)^{\frac{t_1}{m}-p\gamma_1}$, $z_j \in D$ for some t_1, where s is large enough.

The theorem has been proved.

Below, we provide another new sharp trace theorem for the Bloch-type analytic function spaces in the pseudoconvex domains with a smooth boundary in $\mathbb{C}^n$.

Let us assume that

$$A_{log}(r_1,\dots,r_m) = \{f \in H(D^m):$$

$$\sup_{\substack{z_j \in D \\ \sum_{j=1}^{m}\frac{1}{r_j}=1}} |f(z_1,\dots,z_m)| \prod_{j=1}^{m} (log\frac{c}{\delta(z_j)})^{-\frac{1}{r_j}} \delta(z_j)^{1/r_j} < \infty\},$$

$$j=1,\dots,m, \qquad r_j>0, \qquad c = diam(D),$$

$$A_{\vec{\tau}}^{\infty}(D^m) = \{f \in H(D^m): \sup_{\substack{z_1 \in D \\ \dots \\ z_m \in D}} |f(z_1,\dots,z_m)| \prod_{j=1}^{m} \delta(z_j)^{r_j} < \infty\},$$

$$A_r^{\infty}(D) = \{f \in H(D): \sup_{z\in D} |f(z)|\delta(z)^r < \infty\},$$

$r>0$, $r_j>0$, $j=1,\dots,m$.

It can be shown that the spaces introduce above are the Banach spaces in D and in D^m; it can also be shown that these classes contain a large enough τ and r_j the classical weighted Bergman spaces A_α^p for some α and for each $p \geq 1$ in D and in D^m. In addition, such type spaces were studied by many authors during the last several decades in the domains of various type in one and higher dimension (see, for example [1, 2, 15, 6, 7, 3] and various references also there).

Theorem 2.

1) *Suppose* $r_j>0$, $j=1,\dots,m$, $\sum_{j=1}^{m} r_j = r$. *Then,*

$$Trace\ (A_{\vec{r}}^{\infty})(D^m) = A_r^{\infty}(D).$$

2) *Suppose* $r_j>0$, $\sum_{j=1}^{m} r_j = 1$, $j=1,\dots,m$. *Then,*

$$Trace\ A_{log}(r_1,\dots,r_m) = A_{log}(1)$$

where

$$A_{log}(1) = \{f \in H(D): \sup_{z\in D} |f(z)|\delta(z)(log\frac{c}{\delta(z)})^{-1} < \infty\}.$$

Remark 4. *The theorem 2 is obvious for $m = 1$ case. For the unit ball, it can be seen in [2], [4]. Arguments we use here are rather similar to the unit ball proof.*

The proof of the Theorem 2. Let us prove the first case now.

We have $F(z, \dots, z) = f(z)$, $z \in D$, where $f \in A^{\infty}_{\sum_{k=1}^{m} r_j}$ (hence, $f \in A^1_t$ for a large t) and

$$F(z_1, \dots, z_m) = C_{\tilde{\beta}} \int_D f(w)(\delta(w))^{\tilde{\beta}} \left(\prod_{j=1}^{m} K_{\frac{s}{m}}(w, z_j) \right) dv(w) \qquad (3)$$

for a large enough $\tilde{\beta}$ and $s = \tilde{\beta} + n + 1$, $z_j \in D$, $j = 1, \dots, m$, where $C_{\tilde{\beta}}$ is the Bergman representation constant. This follows directly from the definition 1 and lemma 2.

Then it is obvious that $Trace\ (A^{\infty}_{\vec{r}}) \subset A^{\infty}_{r}(D)$. Since, we have

$$\sup_{z \in D} |f(z, \dots, z)|(\delta(z))^{\sum_{j=1}^{m} r_j} \leq \sup_{\substack{z_1 \in D \\ \dots \\ z_m \in D}} |f(z_1, \dots, z_m)| \prod_{j=1}^{m} (\delta(z_j))^{r_j},$$

$r_j > 0$, $j = 1, \dots, m$.

We also have ([17], [6], [7])

$$\int_D |K_{n+t}(z, \zeta)| \delta(z)^{s-1} dv(z) \leq C_{s,t} \delta(\zeta)^{s-t},\ \zeta \in D, \qquad (4)$$

$$\int_D |K_{n+t}(z, \zeta)| \delta(\zeta)^{s-1} dv(\zeta) \leq C_{s,t} \delta(z)^{s-t},\ z \in D \qquad (5)$$

where K is a kernel of $n + t$ type, $s > 0$, $s - t < 0$, $t > 0$.

To prove the reverse assertion, we use the Holder's inequality for m functions from (3), the Forelly–Rudin type estimates (4), (5) and the statement that the product of m kernels of t type is a mt type kernel.

$$|F(z_1, \dots, z_m)| \leq C_1 \|f\|_{A^{\infty}_{\vec{r}}} \int_D (\delta(w))^{\tilde{\beta} - \sum_{j=1}^{m} r_j} \times$$

$$\times \left| \prod_{j=1}^{m} K_{\frac{\beta + \tilde{n} + 1}{m}}(w, z_j) \right| dv(w) \leq$$

$$\leq C_2\|f\|_{A^\infty_{\vec{r}}}\prod_{j=1}^{m}\left(\int_D(\delta(w))^{-r_jm+\tilde{\beta}}|K_{\frac{\tilde{\beta}+n+1}{m}}(w,z_j)|^m dv(w)\right)^{\frac{1}{m}}\leq$$

$$\leq\frac{C_3}{(\delta(z_1))^{r_1}\cdots(\delta(z_m))^{r_m}};\ r_j>0,\ z_j\in D,\ j=1,\dots,m$$

for a large enough $\tilde{\beta}$ and the first part of our theorem has now been proved.

The proof of the second case repeats the proof of the theorem on traces of $A(r_1,\dots,r_m)$ spaces and the only new ingredient is the following vital assertion in D which can be found, for example, in [18].

Let K be a kernel of type n+1+t and let $-1<s<t$. Set $D_0=$ $=diam(D)$. Then, for every $k\in\mathbb{N}$ we have the following estimate

$$\int_D|K(z,\tau)|\delta(\tau)^s(\log^k\frac{D_0}{\delta(\tau)})dv(r)\leq\frac{c}{\delta(z)^{t-s}}\log^k\frac{D_0}{\delta(z)},\ z\in D$$

for some constant c, $c>0$.

The theorem 2 has been proved.

Remark 5. *For $D=B$ (unit ball) the theorem 2 is known (see [2]).*

Note that the traces of the mixed norm Bergman spaces on D^m analogues of the Bergman type classes in the polyball, that are the analytic spaces with the mixed norms $\|f\|_{A^{\vec{p}}_{\vec{\alpha}}}$

$$A^{\vec{p}}_{\vec{\alpha}}(D^m)=\left\{f\in H(D^m),\|f\|_{A^{\vec{p}}_{\vec{\alpha}}}=\left(\int_D\cdots\left(\int_D|f(w_1,\dots,w_m)|^{p_1}\times\right.\right.\right.$$

$$\left.\left.\times\delta(w_1)^{\alpha_1}dv(w_1)\right)^{p_2/p_1}\dots\right)^{\frac{1}{p_m}}<\infty\Bigg\},$$

for $p_{i+1}\geq p_i$, $p_i\geq1$, $\alpha_i>-1$, $i=1,\dots,m$, can be also calculated with the help of the approaches, developed in this paper, namely the following result is valid.

Theorem 3. *Let $p_{i+1}\geq p_i$, $p_i\geq1$, $i=1,\dots,m$. Let*

$$\gamma=\alpha_m+\sum_{j=1}^{m-1}(n+1+\alpha_j)\frac{p_m}{p_j},$$

$\alpha_j > -1$, $j = 1, \dots, m$. *If* $f \in A^{p_1,\dots,p_m}_{\alpha_1,\dots,\alpha_m}(D^m)$ *then*

$$\int_D |f(z,\dots,z)|^{p_m} dv_\gamma(z) \le c\|f\|_{A^{\vec{p}}_{\vec{\alpha}}}.$$

Moreover $Trace\ A^{\vec{p}}_{\vec{\alpha}}(D^m) = A^{p_m}_\gamma(D)$, $1 \le p_j < \infty$, $j = 1, \dots, m$.

Remark 6. *The case of the unit ball was considered in [4], this theorem for this case was presented there.*

We finally define the Besov type analytic function spaces on $D^m \subset \mathbb{C}^n$ as follows

$$A^p_{\delta,\vec{k}}(D^m) = \{f \in H(D^m) : \left(\sum_{|\alpha_1| \le k_1} \cdots \sum_{|\alpha_n| \le k_m} \int_D \cdots \int_D |\mathcal{D}^{\alpha_1 \dots \alpha_n} f|^p \times \right.$$

$$\left. \times (-\rho_1)^{\delta-1} \dots (-\rho_m)^{\delta-1} dv(\zeta_1, \dots, \zeta_m)\right)^{1/p} < \infty,$$

$$\rho_j(z) = \rho(z_j), j = 1, \dots, m, \qquad 0 < p < \infty, \qquad \delta > 0.$$

Based on the lemma 8 and following the proof of the Theorem 1, we can show the following assertion repeating arguments of proof we provided earlier for the unit ball:

Theorem 4. $A^p_{mt+m(n+1)-(n+1)-\sum_{j=1}^m k_j}(D) \subset (Trace)(A^p_{t+1,\vec{k}})$; *for* $p \le 1$, $t > t_0$ *where* t_0 *is large enough.*

Using lemma 3, some not sharp assertions of this type can also be proved.

We finally add an interesting comment. It will be interesting to study and then to calculate the Bergman integral operators, traces of the Bergman-type spaces of the analytic functions $f(z_1, \dots, z_s)$ of mixed type with finite quazinorms

$$\left(\prod_{k=1}^s \int_{X_k} |f(\underbrace{z_1, \dots, z_m}_{m}, \underbrace{z_{m+1}, \dots, z_{m+k_1}}_{k_1}, \dots, z_s)|^p \prod_{k=1}^s dv_k(z_k)\right),$$

$0 < p \le \infty$, where X_i, $i = 1, \dots, r$ is a general Siegel domain of second type (either pseudoconvex or tubular domains), we assume that

$$X_1 = \dots X_m = X_1,\ X_{m+1} = \dots X_{m+k_1} = X_2,\ \dots X_{s-k_r} = \dots X_s = X_r,$$

$$X_1 \neq X_2 \neq \cdots \neq X_r,$$

$r \in \mathbb{N}$, $r > 1$.

Previously only a very special case was considered when $X_1 = \dots X_r = X$, $1 \leq p \leq \infty$ where X is a tubular domain over symmetric cone, or bounded strongly pseudoconvex domain with a smooth boundary.

Complete solutions for this and even distance type problems (see [20], [14]) even in this general case may be also obtained by similar techniques, which we used above, based on r-lattices. The arguments may be similar. The key for the solution is the Whitney type decomposition of each $\{X_i\}$ space and properties of the analytic functions on them and Forelly–Rudin-type estimates we used above for each $\{X_i\}$ type space and the analytic function $f(\dots, z_i, \dots)$ on $\{X_i\}$, in each variable separately. We leave this procedure for the interested readers. The results of this paper partially may be extended to bounded minimal homogeneous domains based on recent subtle results of Yamaji (see for example [21] and references there) related to estimates of the Bergman kernel and properties of r-lattices on these domains. In these papers, complete analogues of some of our lemmas 2-8 can be seen. These domains are rather general, in particular they serve as direct extentions of so-called well studied bounded symmetric domains in higher dimension.

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Bryansk State University
14 Bezhitskaya st., Bryansk 241036, Russia
E-mail: rshamoyan@gmail.com, SergKurilenko@gmail.com